II International conference
Multiscale methods and Large-scale Scientific Computing

Automatic Time Step Selection for Numerical Solution of Neutron Diffusion Problems

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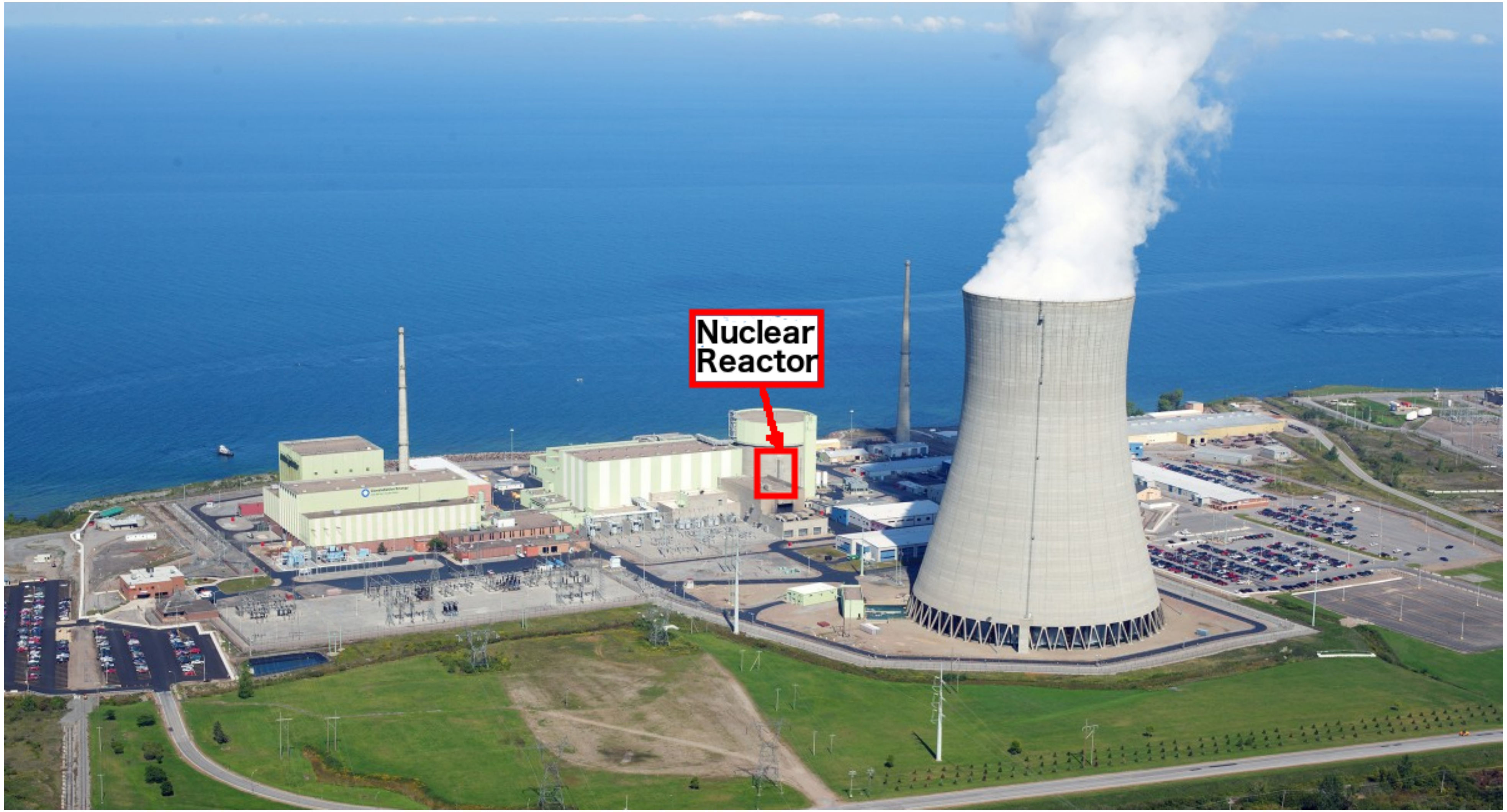
Nuclear power station



Nuclear reactor

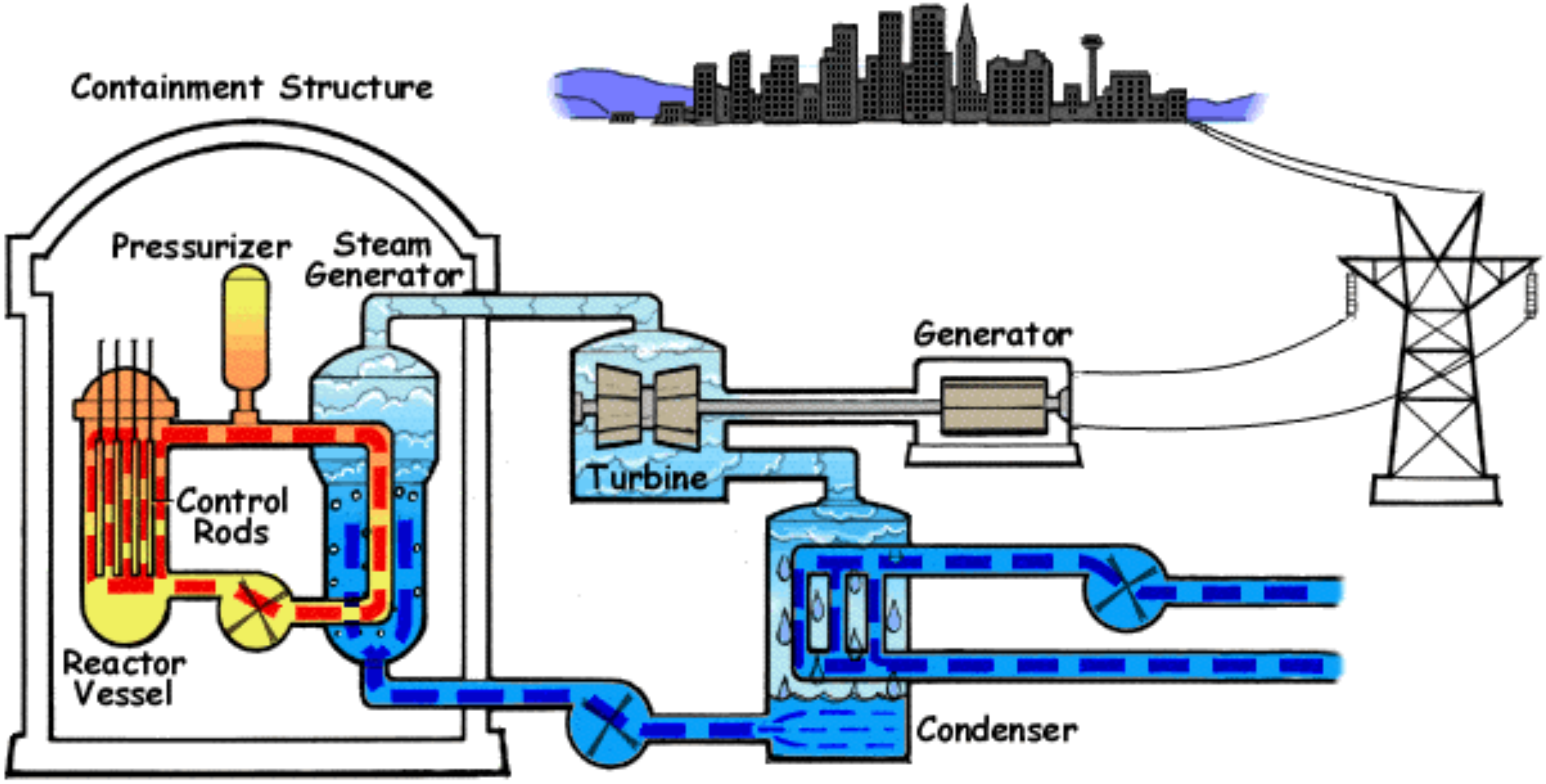
Steam turbine

Generator

Cooling system

Other systems

Nuclear power station



Math modeling

- Neutron transport equation (time, energy, spatial and angular variables)
- Mutigroup diffusion equation

Time step control

- Cauchy problem

  The problem of the time step control is relatively well developed for the Cauchy problem solution of differential equations systems (Ascher 1998, Gear 1971, Hairer 1987). The basic approach is to use additional calculations at a new time step to estimate the approximate solution.

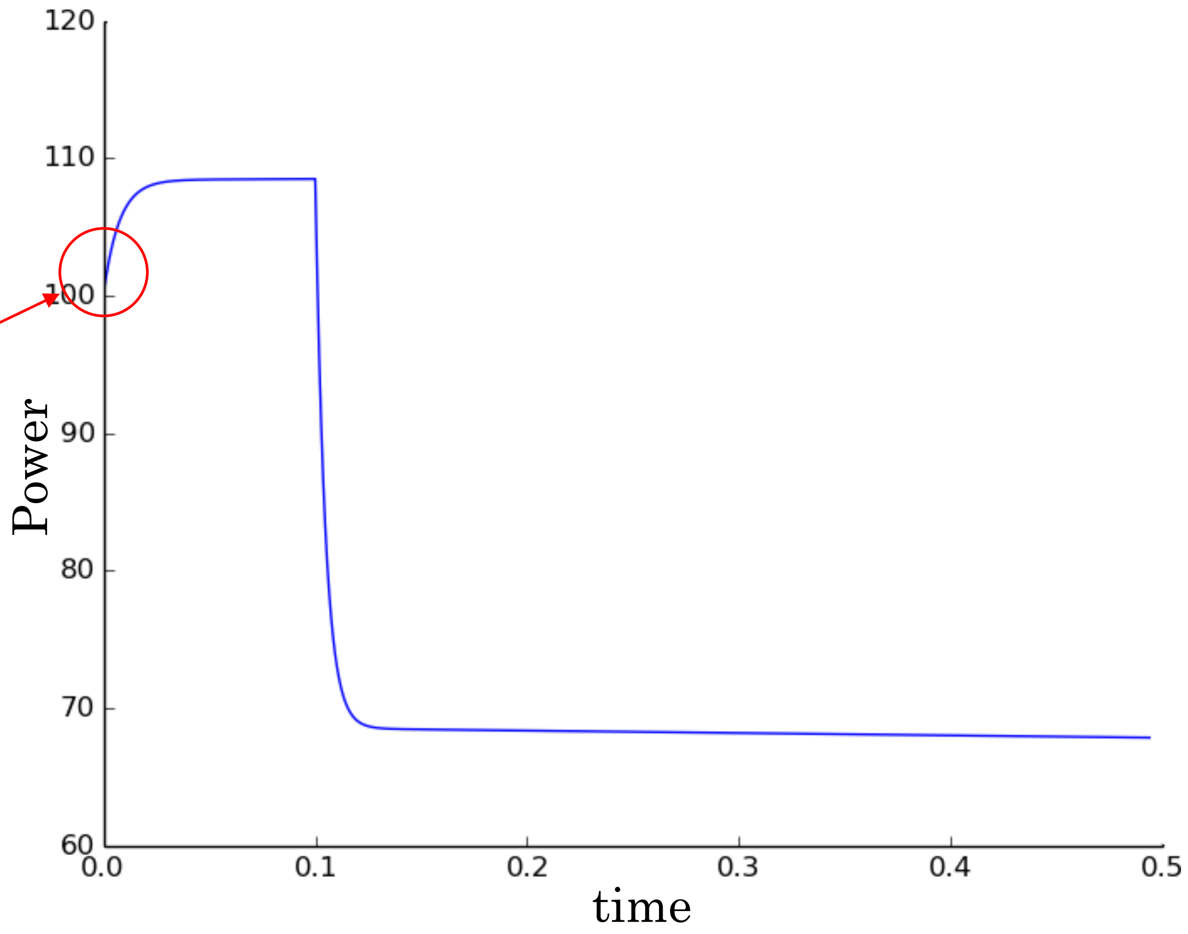
- Additional calculation

  Additional calculations for estimating the error of the approximate solution can be carried out in different ways. The best-known strategy is connected with the solution of the problem on a separate time interval using the given step (the first solution) and with a step two times smaller (the second solution)

- A posteriori

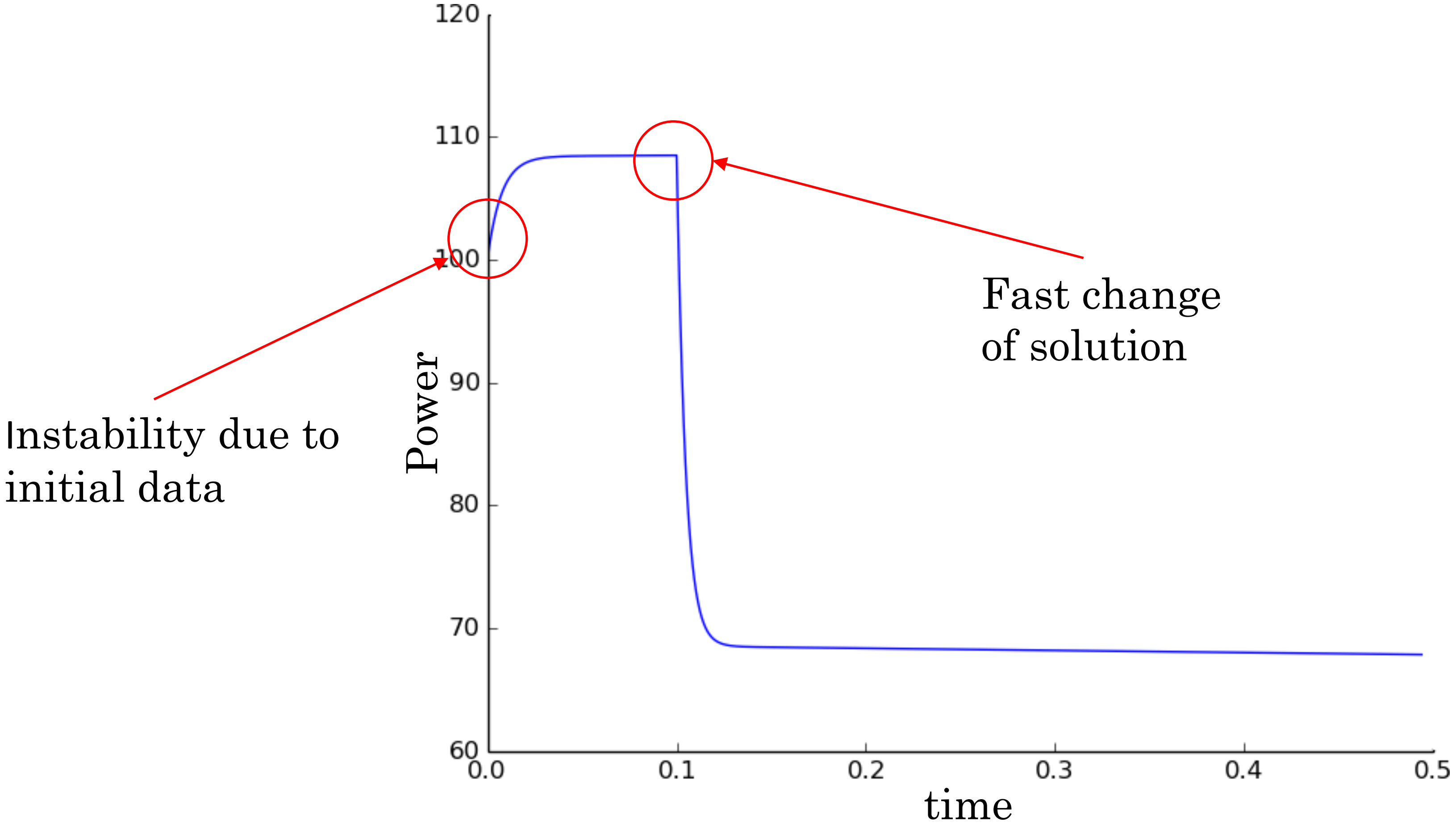
  This way of selecting the time step is related to the class of a posteriori accuracy estimation methods. The decision as to suitable the time step or the re-calculation is accepted only after the calculation is completed.

Motivation



Instability due to
initial data

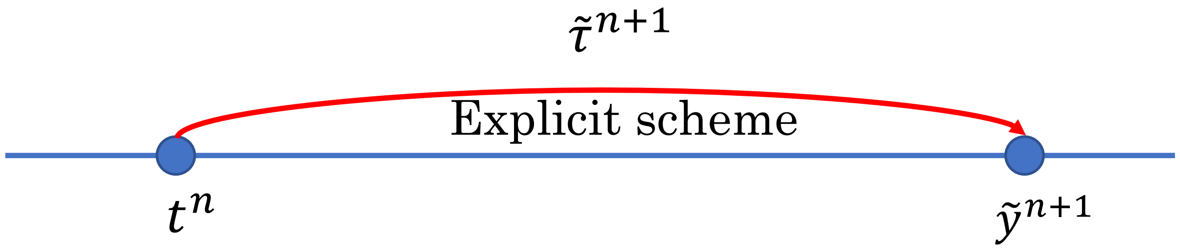
Motivation



120
110
100
90
80
70
60
Power
0.0
0.1
0.2
0.3
0.4
0.5
time
Instability due to initial data
Fast change of solution

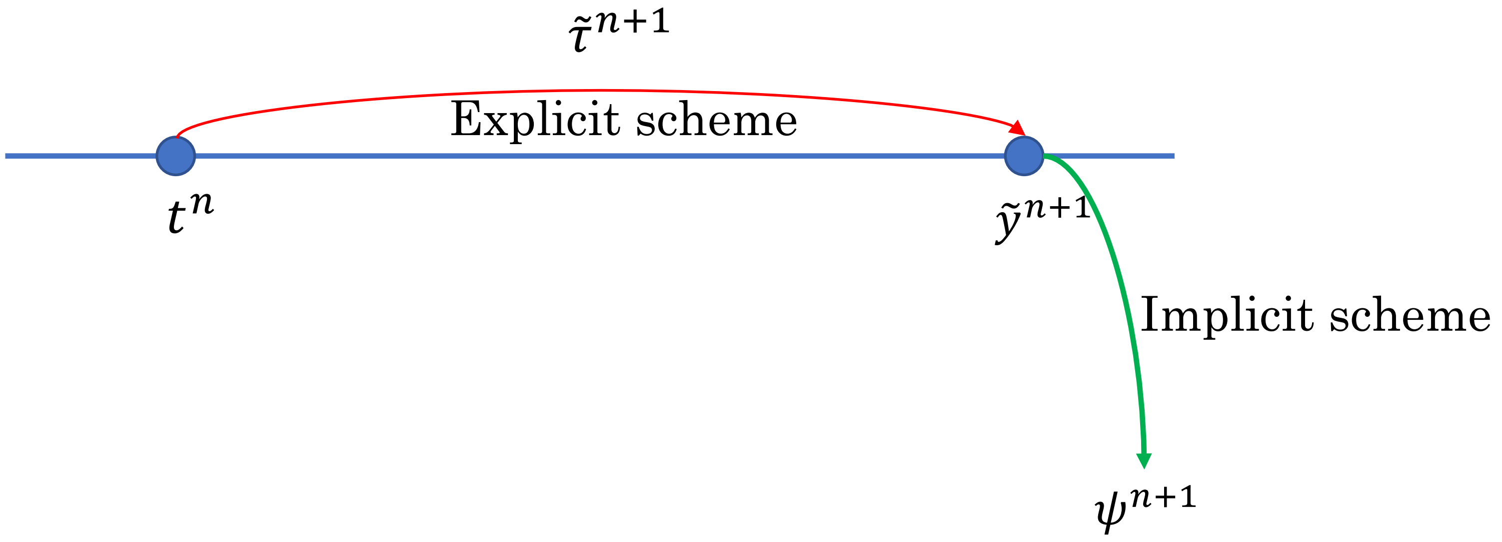
Algorithm

$\tilde{\tau}^{n+1} = \gamma\tau^{n}, \ \gamma = 1.5$



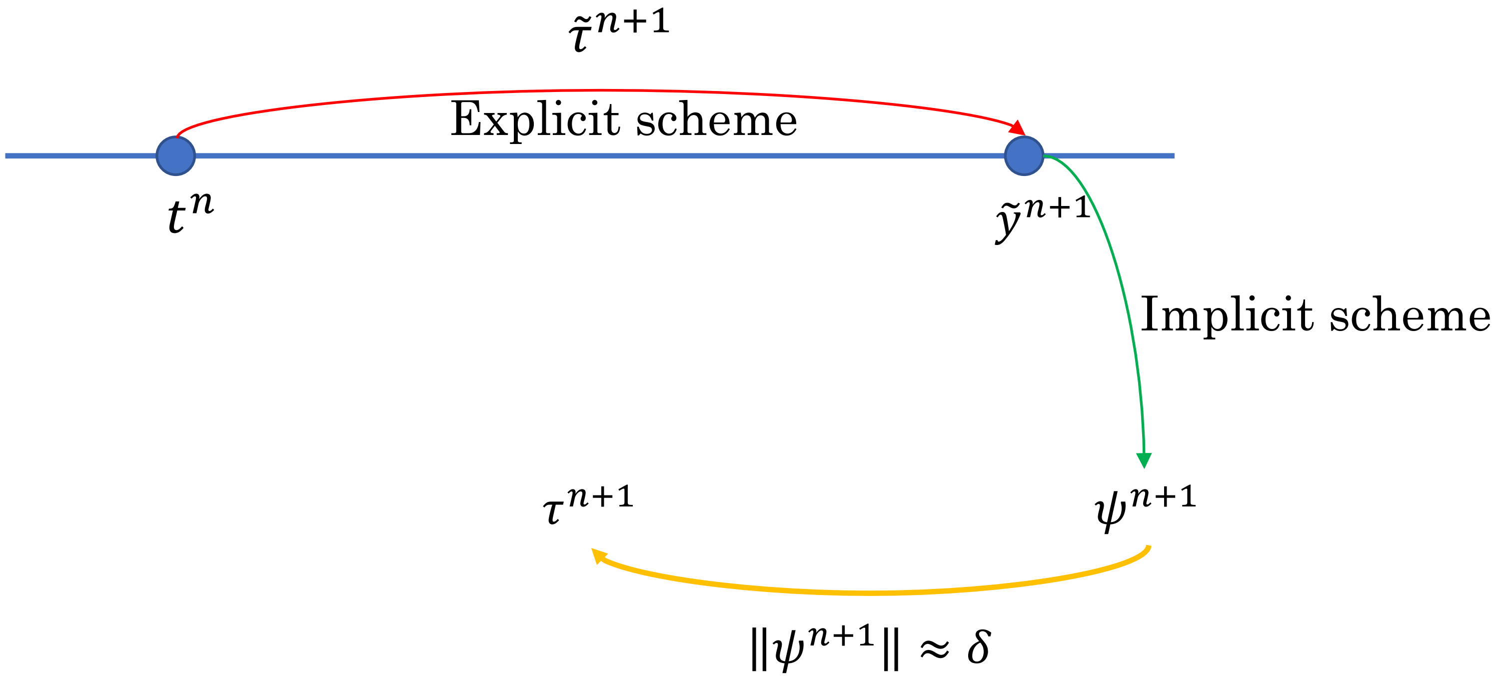
Algorithm

ψ^{n+1} - approximation error



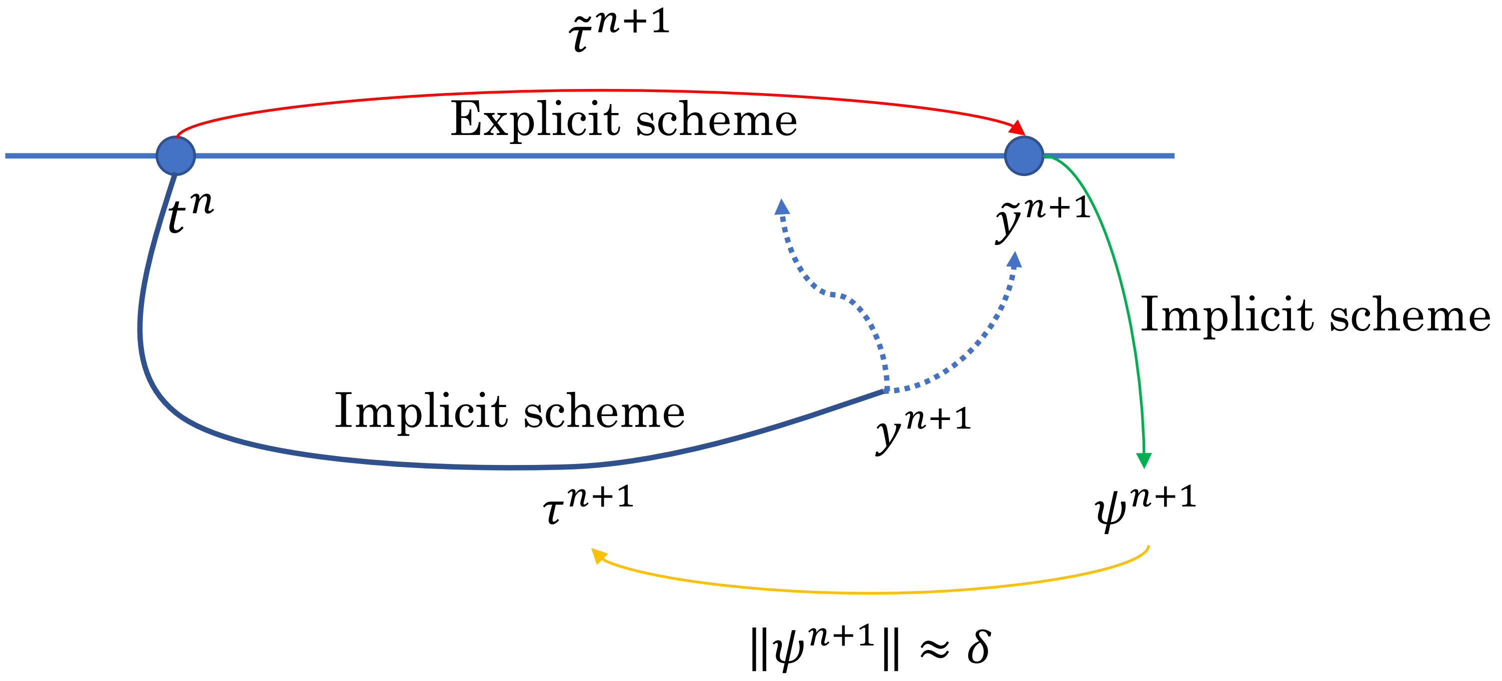
Algorithm

δ- error parameter



Algorithm

$y^{n+1}, t^{n+1} = t^n + \tau^{n+1}$



Neutron diffusion equation

Diffusion approximation

$$\frac{1}{v}\frac{\partial \phi}{\partial t} - \nabla \cdot D\nabla\phi + \Sigma_a\phi = (1-\beta)\nu\Sigma_f\phi + \lambda c,$$
$$\frac{\partial c}{\partial t} + \lambda c = \beta\nu\Sigma_f\phi.$$

Boundary condition

$$D\frac{\partial \phi}{\partial n} + \gamma_a\phi = 0.$$

Initial condition

$$\phi(0) = \phi^0,\ c(0) = c^0.$$

Calculated formulas

The needed time step

$$\tau^{n+1} = \max\{\tau^0, \min\{\gamma_{n+1}, \gamma\}\tau^n\}.$$

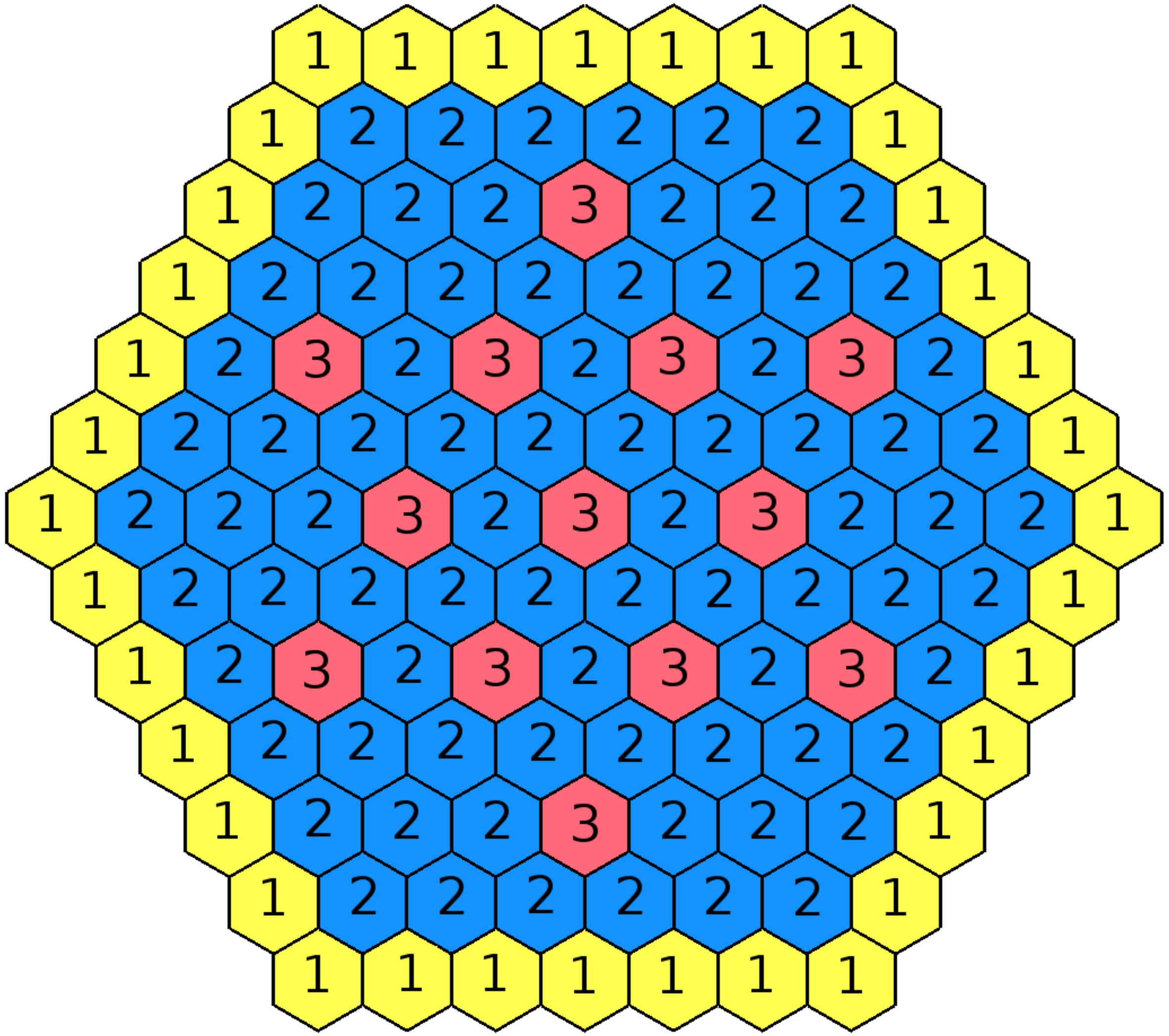
Calculated formula

$$\gamma_{n+1} = \frac{\delta}{\|(A^{n+1} - A^n)\boldsymbol{\varphi}^{\boldsymbol{n}} + A^{n+1}(\widetilde{\boldsymbol{\varphi}}^{\boldsymbol{n+1}} - \boldsymbol{\varphi}^{\boldsymbol{n}})\|}\gamma.$$

Where $A \geq 0$:

$$\boldsymbol{\varphi} = \{\varphi, s\}, \quad A = \begin{pmatrix} -\nabla \cdot D\nabla + \Sigma_a - (1-\beta)\nu\Sigma_f - \lambda & 0 \\ 0 & \lambda - \beta\nu\Sigma_f \end{pmatrix}.$$

IAEA-2D benchmark



Without reflector

One group of *instantaneous* and *delayed* neutrons

Modeling effect of immersion or extraction of control rods

Scenario

Define the scenario of process

1. The spectral problem is solved (initial condition)
2. Calculation for the nonstationary model in the range 0 to 0.5 sec
3. At a moment of 0.1 sec the value Σ_a for the zone 3 changes to ± 0.000625

Software

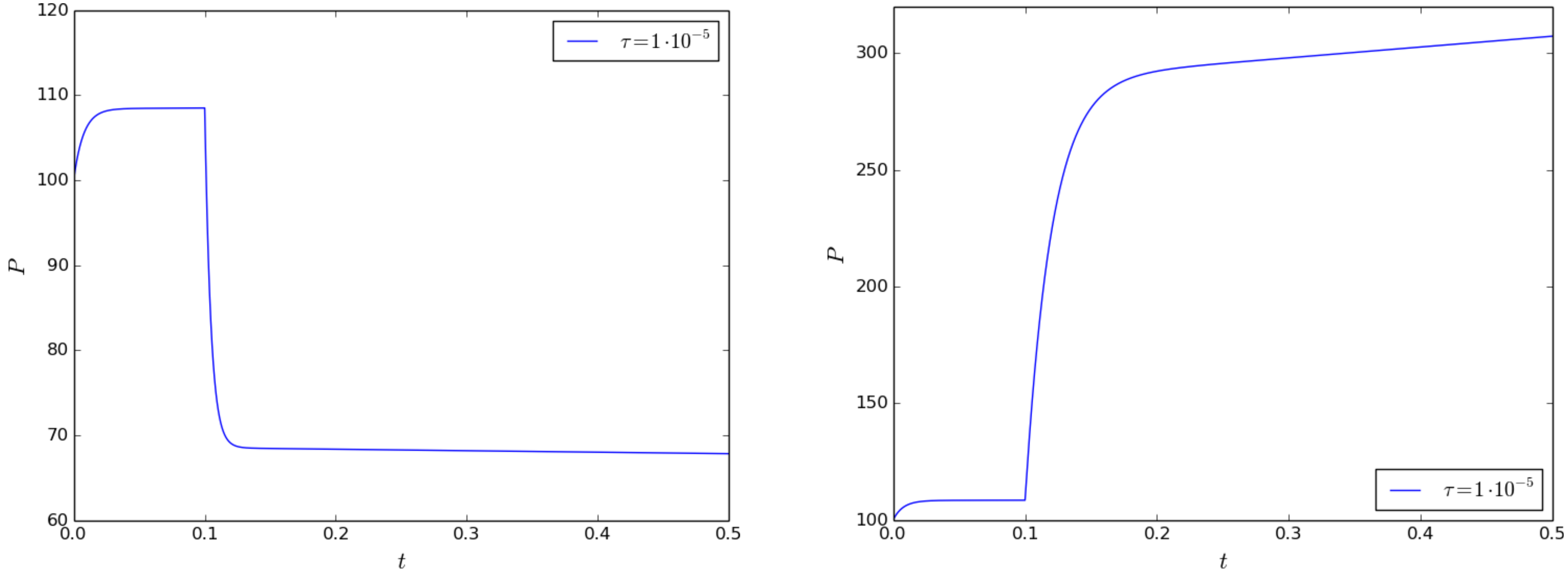
gmsh

SLEPc

FEniCS project

python™

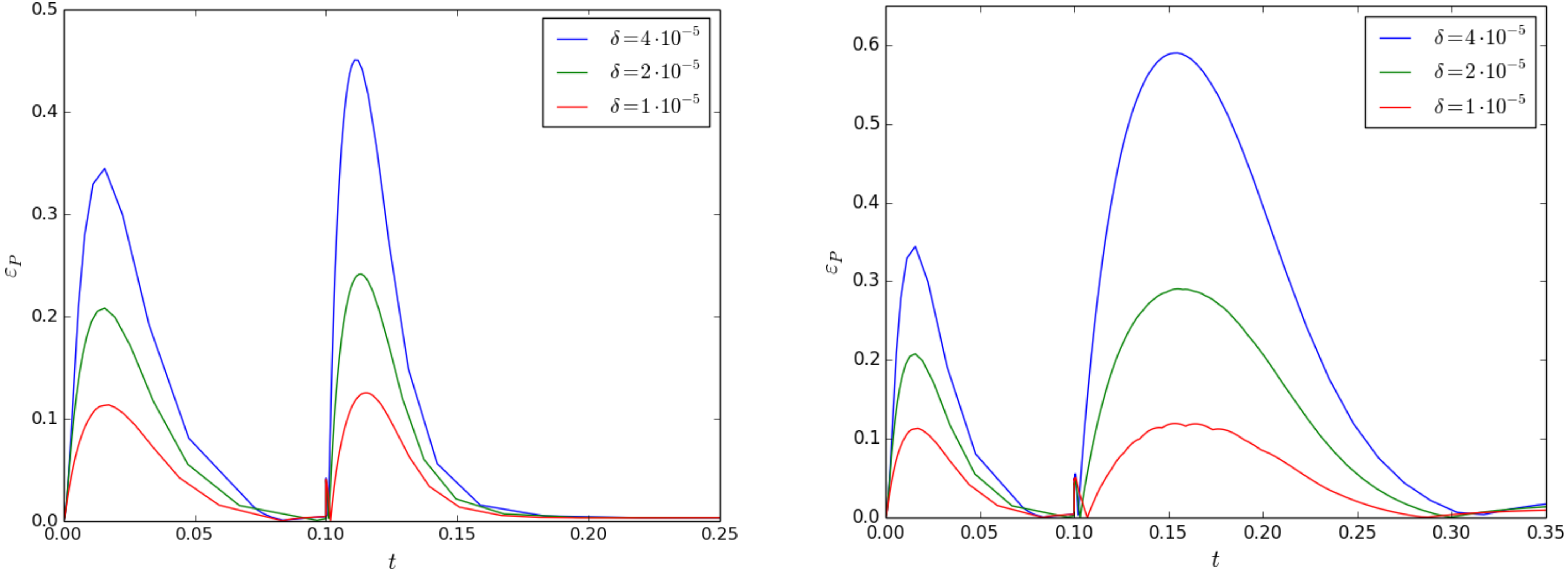
Nuclear power



Nuclear power for immersion (left) and extraction (right)

$$P(t) = a \sum_{g=1}^{G} \int_{\Omega} \Sigma_{fg} \phi_g \, d\boldsymbol{x}$$

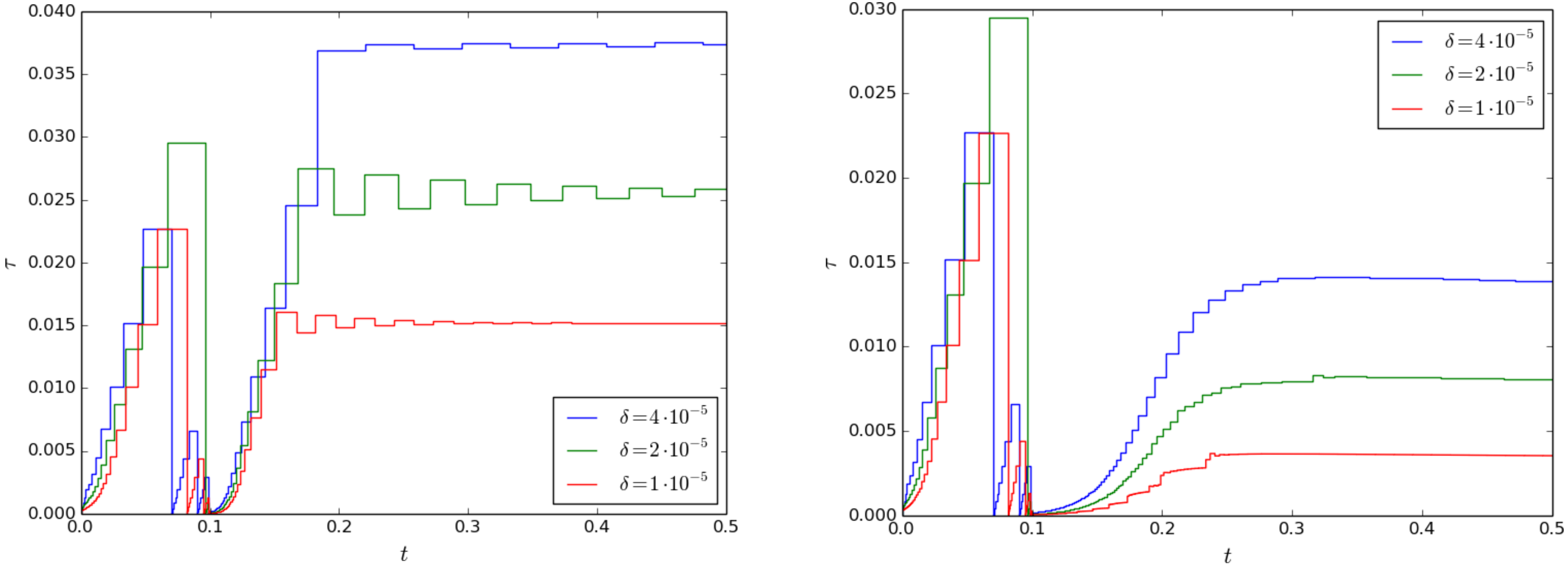
Error



Error for immersion (left) and extraction (right)

$$\epsilon_P(t) = |P_{ref} - P|$$

Time step



Time steps for immersion (left) and extraction (right)

Counting time and number of steps

	immersion			extraction		
δ	$\max(\epsilon_P)$	n	t, sec	$\max(\epsilon_P)$	n	t, sec
$4 \cdot 10^{-5}$	0.450	136	16	0.590	241	35
$2 \cdot 10^{-5}$	0.241	159	20	0.290	373	62
$1 \cdot 10^{-5}$	0.125	270	37	0.120	773	145

Reference solution: fixed time step is 10^{-5}, number of steps is 50000, counting time is 2130 sec.

Conclusion

- An algorithm for automatic time step selection for numerical solution of neutron diffusion problems has been developed.

- The solution is obtained using guaranteed stable implicit schemes, and the step choice is performed with the use of the solution obtained by an explicit scheme.

- Calculation results obtained for a neutron diffusion problems demonstrate reliability of the proposed algorithm for time step choice.

Thank you for your attention!