II International conference
**Multiscale methods and**
**Large-scale Scientific Computing**

Kazan Federal
UNIVERSITY

# Superelement Method of Multiscale Modeling of Petroleum Reservoirs

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# Motivation

Forecast and control of the full range of oil production processes

⇩

Detailed description of the parameters of flow in oil reservoir

⇩

Mathematical and numerical modeling of multiphase flows

Grid step for detailed geological models

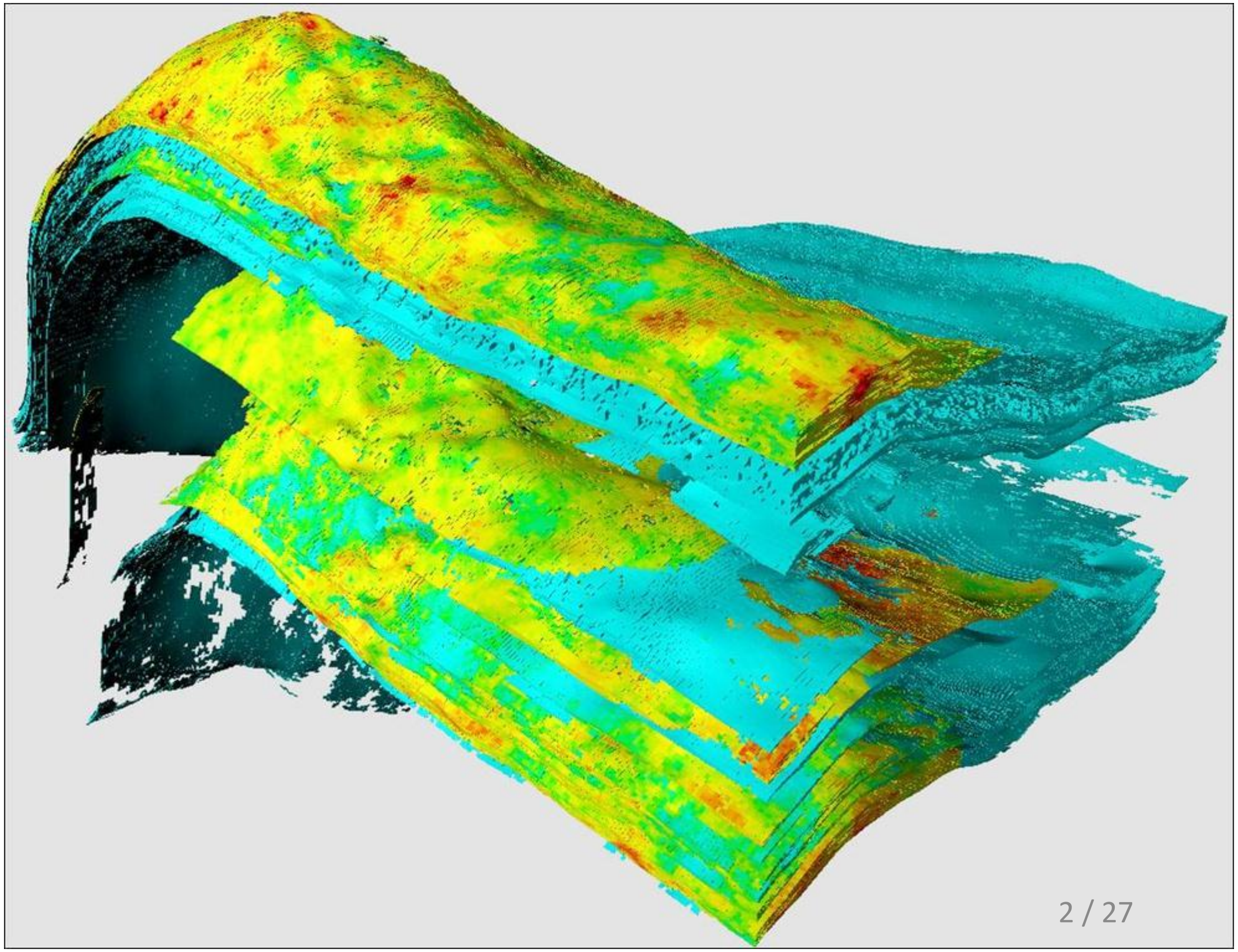
1 m (XY) / 0.1 m (Z)

Typical reservoir dimensions

$10^3$-$10^4$ m (XY) / 10-$10^2$ m (Z)

Dimension of grids

$10^8$-$10^9$



# The current state

## 1. TRANSITION TO COARSE GRIDS

- strict mathematical apparatus of the reservoir properties averaging

Bakhvalov N., Belyaev A., Bensoussan A., Berditchevsky A., Cherkaev A., Kozlov S., Lions J.L., Lurie K., Panfilov M., Papanicolaou G., and etc.

- approximate upscaling of the reservoir properties

Aasen J., Arbogast T., Berry D., Bulygin V., Celia M., Christie M., Coats K., Dale M., Durlofsky L., Efendiev Y., Ekrann S., Gasda S., Hearn C., Holden L., Hou T.Y., Jacks H., Kanevskaya R., Kurbanov A., Kyte J., Lenormand R., Nielsen B., Rodionov S., Stone H., Wang K., Wu X.H., and etc.

## 2. MULTISCALE MODELING

- basis functions; pressure on a coarse grid; saturation on a fine grid

Aarnes J., Blunt M., Chen Z., Efendiev Y., Fedorenko R., Ginting V., Gautier Y., Hou T., Jenny P., Lee S., Pergament A., Semiletov V., Strakhovskaya L., Tchelepi H. , Tomin P., Wu X.H., and etc.

## 3. SECTORAL MODELING

- an unified methodology for numerical modeling from global to local processes;

Tempest MORE, ECLIPSE, TimeZYX, VIP-Executive, tNavigator, and etc.

- the solution is locally refined on detailed grids; decomposition;

Baryshnikova A., Dolean V., Dzyuba V., Joliet P., Kostyuchenko S., Kudryashova I., Maksimova D., Nataf F., and etc.

**The Aim** - development of a cost-effective and sufficiently accurate method; using at every level of the design of special models with the detail degree appropriate to the scale of the problem.

# The proposed approach – sequential use of special models, determined by the scale and features of the described processes

| Processes | Problems | Models and scales |
|---|---|---|
| **Global dynamics of water flooding.**<br><br>Long-term forecast (decades) | **The general project of an oil deposit development.**<br><br>- general indicators forecast,<br>- reserves production rates analysis,<br>- energy state estimation,<br>- reserves distribution analysis,<br>- problem areas identification. | **Superelement model.**<br><br>Grids resolution (XY / Z)<br>200-500 m / 10-100 m |
| **Interaction of wells on the reservoir section.**<br><br>Medium-term forecast (month - year) | **The project of a section development.**<br><br>- oil reserves localization,<br>- interaction of wells evaluation,<br>- selection of the wells for treatments. | **3D model of medium resolution.**<br><br>Grids resolution (XY / Z)<br>10-50 m / 1 m |
| **Local processes near wells.**<br><br>Short-term forecast (hour - month) | **The design of well treatments.**<br><br>- local effects description:<br>- *bottomhole zone treatment,*<br>- *horizontal wells,*<br>- *hydraulic fracturing,*<br>- *polymer flooding.* | **High-resolution special models:**<br>*- fixed streamtube model,*<br>*- model of inflow to HF fractures, …*<br><br>Grids resolution (XY / Z)<br>1 m / 0.1 m |

# The initial equations of two-phase flow. Estimations of the scales

- liquid phases are incompressible
- capillary and gravitational forces are negligible

$$\beta \frac{\partial p}{\partial t} + \operatorname{div} \mathbf{u} = 0, \quad m \frac{\partial s}{\partial t} + \operatorname{div}\left( f\, \mathbf{u} \right) = 0$$

$$\mathbf{u} = -\varphi \frac{k}{\mu_w} \nabla p, \;\; \varphi = k_w + \mathrm{K}_\mu k_o, \;\; \mathrm{K}_\mu = \frac{\mu_w}{\mu_o}, \; f = \frac{k_w}{\varphi}$$

$$k_w = S^a, k_o = \left(1 - S\right)^b, S = \frac{s - s_*}{s^* - s_*}$$

The characteristic time scales:

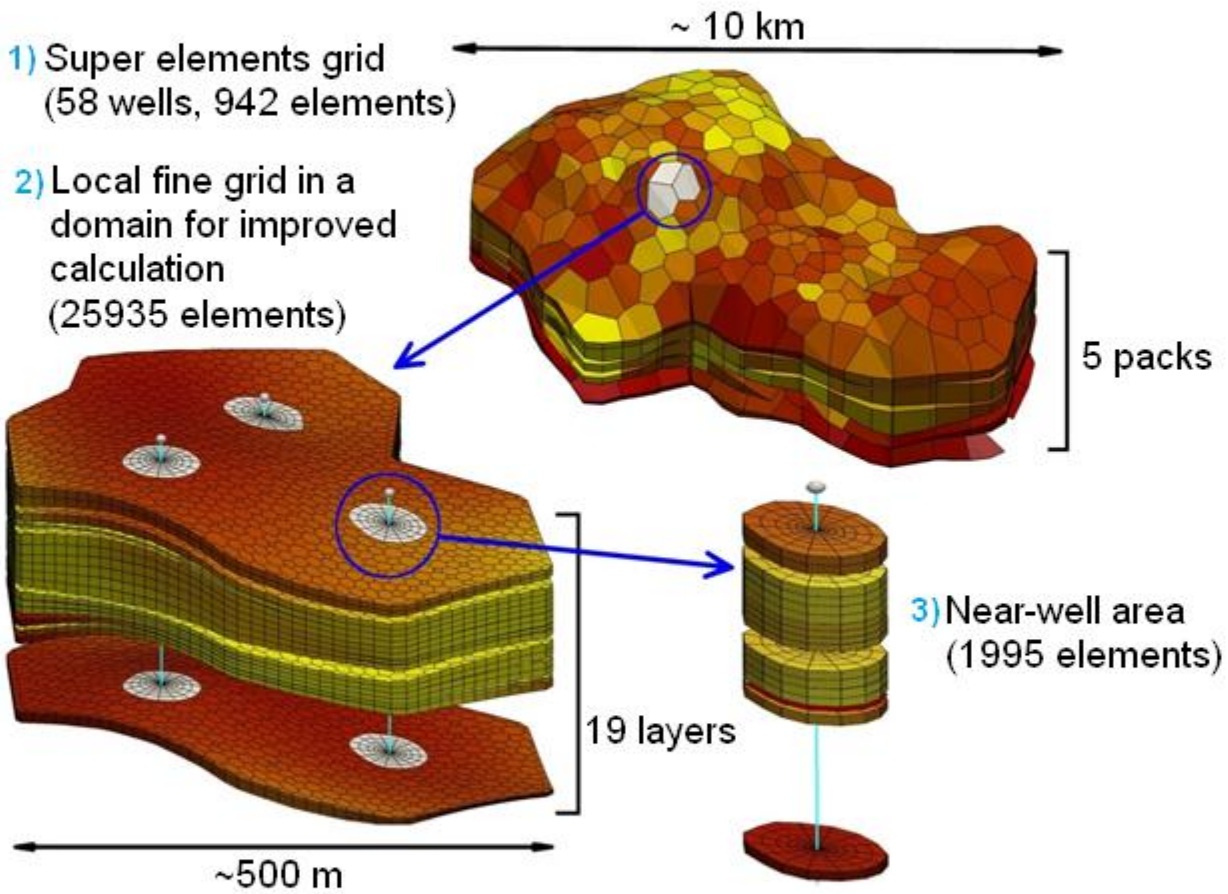
$$t_s = \frac{\mu m^0 L^2}{k \Delta p}, \quad t_p = \frac{\mu m^0 \beta^* L^2}{k}$$

| | | | |
|---|---|---|---|
| 1. Global waterflooding | L ~ $10^2$m | $t_s \sim 10^7$-$10^8$ sec | $t_p \sim 10^4$ -$10^5$ sec |
| 2. Well Interaction | L ~ 10 m | $t_s \sim 10^5$-$10^6$ sec | $t_p \sim 10^2$ -$10^3$ sec |
| 3. The near-wellbore flow | L ~ 1-10 m | $t_s \sim 10^2$-$10^4$ sec | $t_p \sim 10^1$ sec |

# Sequential modeling of the various scales flows

Coarse super element grid → **Cumulative adaptation** of the model

Independent local fine grids → **High accuracy** of the numerical solution near the computational domain's geological and technological features



# Superelement modeling of the global waterflooding

- **The grid dimension should be comparable to the number of wells; grid spacing ≈ distance between wells**

- **Coarse grids are sufficient to describe a smooth average pressure field**

- To describe fast local events, the solution is locally refined

On the boundary of the local refinement region

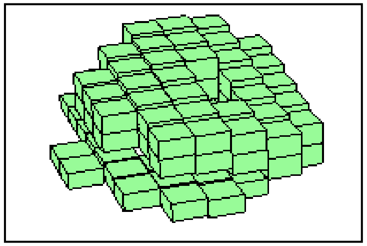
$$\langle p \rangle \approx p, \quad \nabla \langle p \rangle \approx \nabla p$$

## Traditional grids

cell size

↔ 10 - 50 m

↕ 0.1 - 1 m



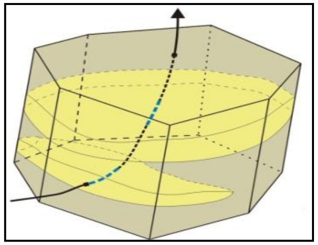
dimension of SLAE ~ $10^6$ - $10^8$

## Super element grid

cell size

↔ 100 - 500 m

↕ 10 - 50 m



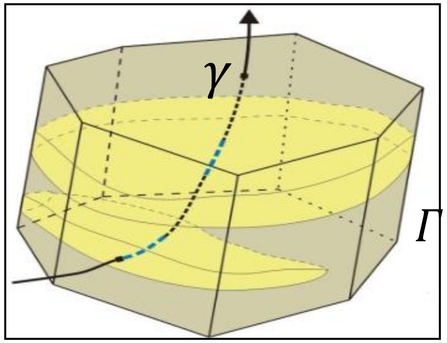
dimension of SLAE ~ $10^2$ - $10^5$

# Accuracy of calculations on a coarse superelement grid

- **problem formulation for smooth average fields** $p$, $s$ ⇨ ✓ elimination of the logarithmic behavior of the function $p$ near the well
- **upscaling of absolute permeability and transition to tensor equations** ⇨ ✓ taking into account the geological structure within the superelement capable of deflecting the flow
- **upscaling of the relative phase permeability functions** ⇨ ✓ taking into account the internal flow structure and the frontal displacement
- **numerical solution according to the FV scheme** ⇨ ✓ construction of a conservative calculation scheme

# Computational scheme construction. Equations for average values

For each superelement (SE) at each time step




**average pressure**

$$\langle p \rangle = \frac{1}{|V|} \int_V p \; dV$$

**average water saturation**

$$\langle s \rangle = \frac{\langle ms \rangle}{\langle m \rangle}$$

$$\beta |V| \frac{\partial \langle p \rangle}{\partial t} + Q_\Gamma + Q_\gamma = 0$$

$$|V| \langle m \rangle \frac{\partial \langle s \rangle}{\partial t} + Q_\Gamma^W + Q_\gamma^W = 0$$

$$Q_\Gamma = \int_\Gamma u_n \, d\Gamma = \sum_j |\Gamma_j| \, \widetilde{u_n}_j \, , \; Q_\gamma = \int_\gamma u_n \, d\gamma = q$$

$$\widetilde{u_n}_j = \frac{1}{|\Gamma_j|} \int_{\Gamma_j} u_n \, d\Gamma \qquad Q_{\Gamma,\gamma}^W = \int_{\Gamma,\gamma} f \, u_n \, d\gamma \, , \qquad Q_\Gamma^W \approx \sum_j |\Gamma_j| \widetilde{f_j \, u_{n j}}, Q_\gamma^W \approx |\gamma| \widetilde{f \, u_n} = \tilde{f} \, q$$

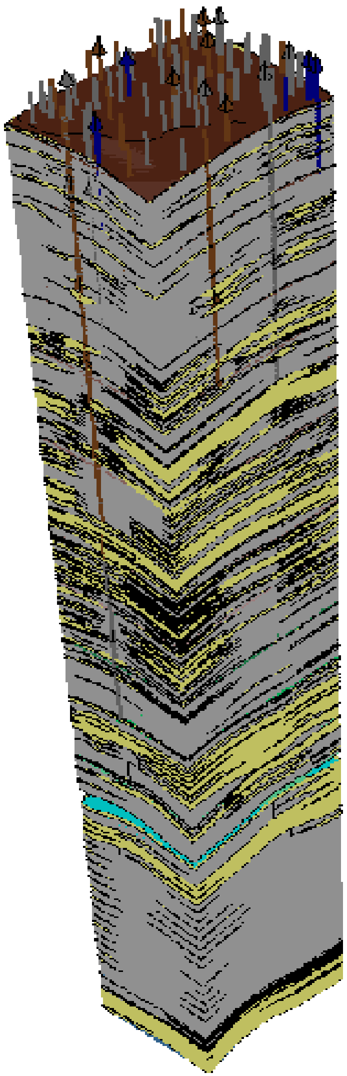
Filling of function $\langle p \rangle \;\; \Rightarrow \;\; \mathbf{U} = -\boldsymbol{\sigma} \cdot \nabla \langle p \rangle \qquad \boldsymbol{\sigma} = \langle \varphi \rangle \mathbf{K} \qquad \mathbf{k} = \mathbf{n} \cdot \mathbf{K}$

Approximation of velocities $\widetilde{u_n} \approx U_n = -(\boldsymbol{\sigma} \cdot \nabla \langle p \rangle) \cdot \mathbf{n} = -\langle \varphi \rangle \mathbf{k} \cdot \nabla \langle p \rangle = -\langle \varphi \rangle |\mathbf{k}| \frac{\partial \langle p \rangle}{\partial k}$

$$\widetilde{f \, u_n} \approx \langle f \rangle \, U_n, \;\; \langle f \rangle = K_w(\langle s \rangle) / \langle \varphi \rangle \qquad \langle \varphi \rangle = \langle k_w(s) + K_\mu \, k_o(s) \rangle \; \approx K_w(\langle s \rangle) + K_\mu \, K_o(\langle s \rangle)$$

# Coarse computational grid

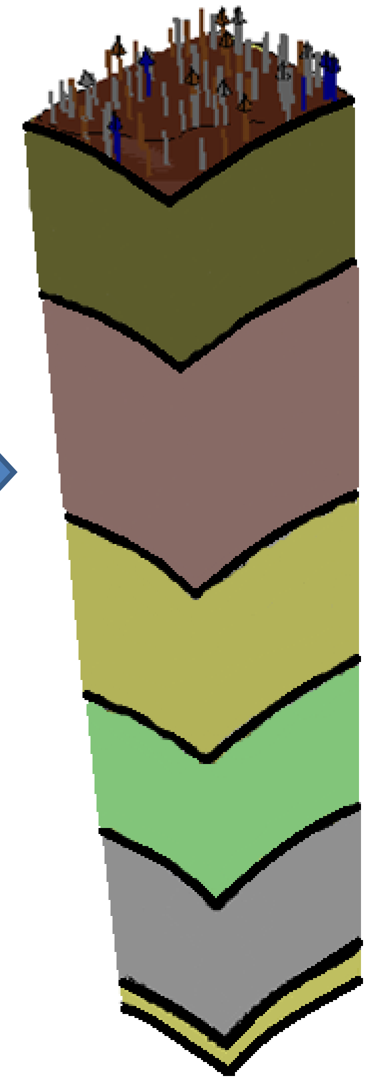
**Characteristic oil formation** – set of sublayers with different properties



Grid coarsening
Upscaling

Absolute permeability tensor

Dynamic pseudo-functions of relative permeability



**Geological grid – thin sublayers**
0.1-1 meters vertically
(layered reservoir)

**Superelement grid**
10-100 meters vertically
(homogenized reservoir)

## The problem of local upscaling of absolute permeability

The effective tensor components ***$K=\{K^{ij}\}$, $i,j=1..3$*** are found in each SE.

Steady state three-dimensional problems of single-phase flow.

Boundary conditions

$$p^b\Big|_{\mathbf{x}\in\Gamma} = \sum_{m=1}^{3} \delta_m^b x_m, \; b = 1..N_b. \; N_b = 3$$

For SE with well

$$\int_\gamma u_n \, d\gamma = q$$

The functional of the deviation of the average normal velocities on the SE faces

$$R^2\left(K^{ij}\right) = \frac{1}{N_b} \sum_{b=1}^{N_b} \rho^b\left(K^{ij}\right) \to \min_{K^{ij}}$$

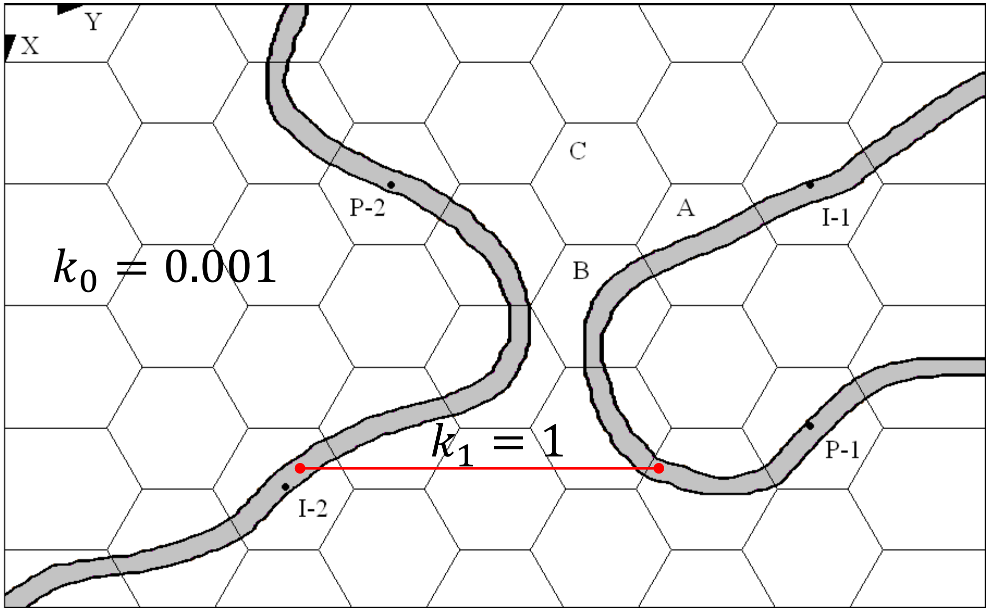
$$\rho^b\left(K^{ij}\right) = \frac{1}{q_*^b} \sum_{k=1}^{N_\Gamma} |\Gamma_k| \left(\tilde{u}_{kn}^b - U_{kn}^b\left(K^{ij}\right)\right)$$

$$q_*^b = \sum_{k=1}^{N_\Gamma} |\Gamma_k| \left|\tilde{u}_{nk}^b\right|$$

# Testing of upscaling techniques. High-permeability channels

Production wells **P-1**, **P-2**

Injection wells **I-1**, **I-2**




**Methods of upscaling**

1) method of volume averaging (MVA),
2) rate averaging method (RAM),
3) minimum dissipation method (MDM),
4) method of superelement upscaling (**MSU**).

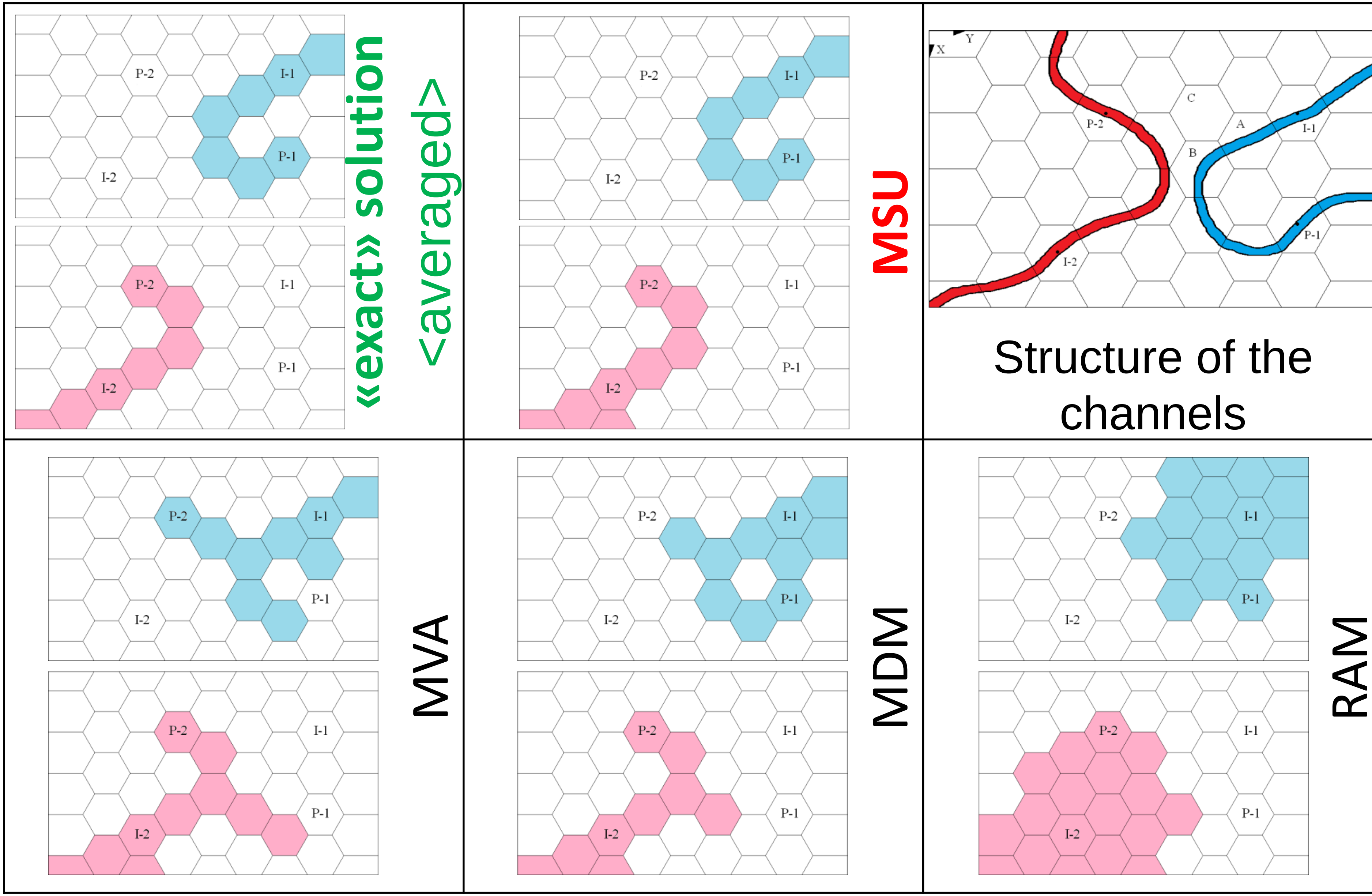
$$r_u = \left\| \frac{u_n - U_n}{u_*} \right\|_2, \quad r_p = \left\| \frac{\langle p \rangle - P}{\Delta P} \right\|_2, \quad R_p = \left\| \frac{\langle p \rangle - P}{\Delta P} \right\|_C$$

$$u_* = \max_k |u_{nk}|, \quad \Delta P = \max_i \langle p \rangle_i - \min_i \langle p \rangle_i$$

| | **MVA** | **RAM** | **MDM** | **MSU** |
|---|---|---|---|---|
| $r_u$ | 0.130 | 0.222 | 0.079 | 0.063 |
| $r_p$ | 0.158 | 5.523 | 0.186 | 0.139 |
| $R_p$ | 0.503 | 18.76 | 0.653 | 0.434 |

# Testing of upscaling techniques. High-permeability channels

Distribution of tracer from injection wells I-1, I-2 at fixed time moment



# The problem of local upscaling of the relative phase permeability functions

**The form of the modified functions of relative phase permeability (MFRP)**

$$K_w(\langle s\rangle) = S^{A(S)}, \quad K_o(\langle s\rangle) = (1-S)^{B(S)}, \qquad S = \frac{\langle s\rangle - s_{\min}}{s* - s_{\min}}$$

$$A(S) = A_0 + A_1 S + A_2 S^2, \quad B(S) = B_0 + B_1 S + B_2 S^2$$

**Coefficients A, B, $s_{\min}$ are found from the condition of minimizing the functional**

$$J\left(\mathbf{A}, \mathbf{B}, s_{\min}\right) = \frac{1}{T}\int_0^T \left(w_1 \Delta Q^2 + w_2 \Delta Q_W^2 + w_3 \Delta q^2 + w_4 \Delta q_W^2\right) dt$$

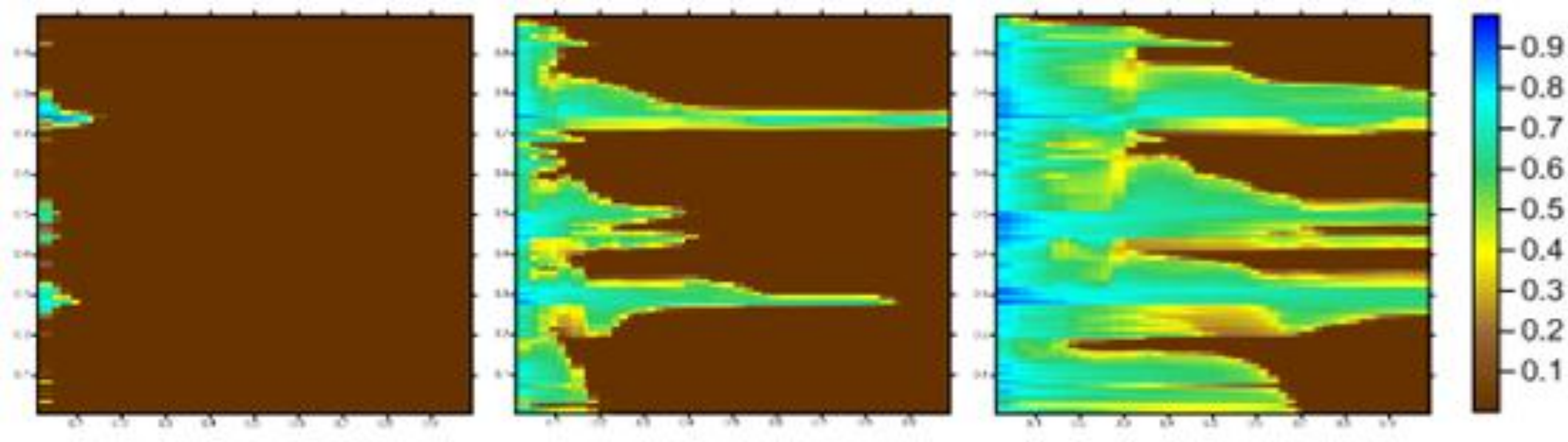
$$\Delta Q = |\Gamma| U_n|_\Gamma - Q_\Gamma, \quad \Delta Q_W = |\Gamma| U_n^W\Big|_\Gamma - Q_\Gamma^W,$$

$$\Delta q = |\Gamma| U_n|_\gamma - Q_\gamma, \quad \Delta q_W = |\Gamma| U_n^W\Big|_\gamma - Q_\gamma^W.$$

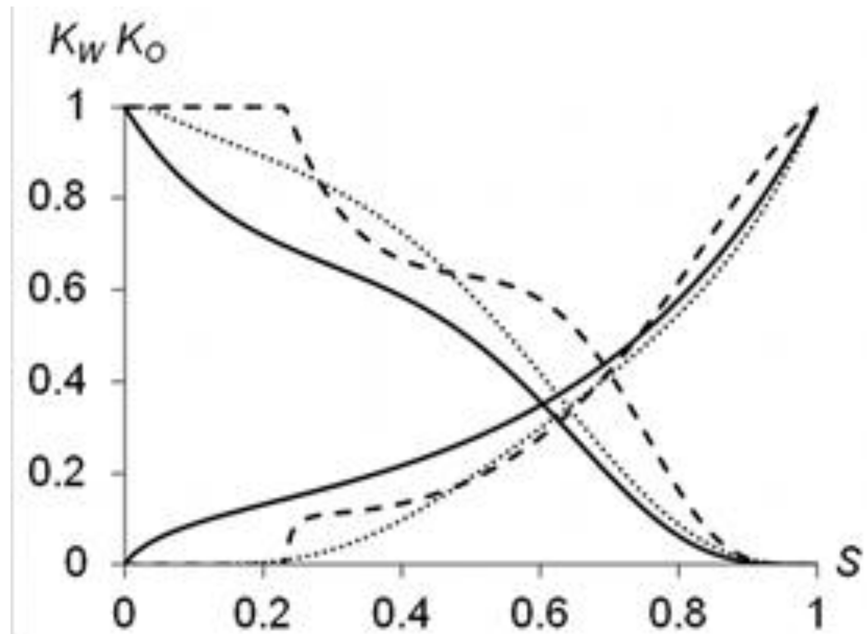
$w_i$ - weight coefficients

| **Waterflooding scenarios** | 1) SE with production well<br>2) SE with injection well<br>3) SE without wells |
|---|---|

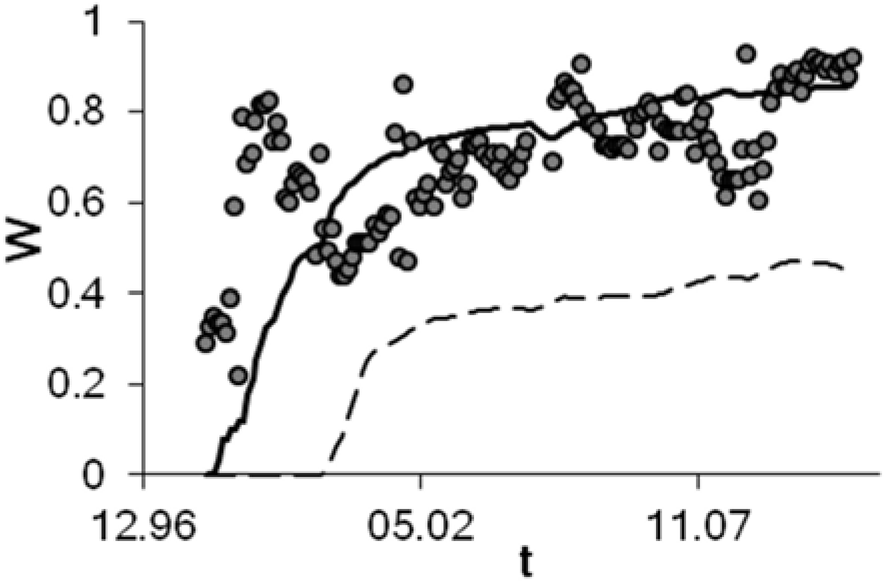
# **Solve results.** Modeling of water flooding of real oil reservoir

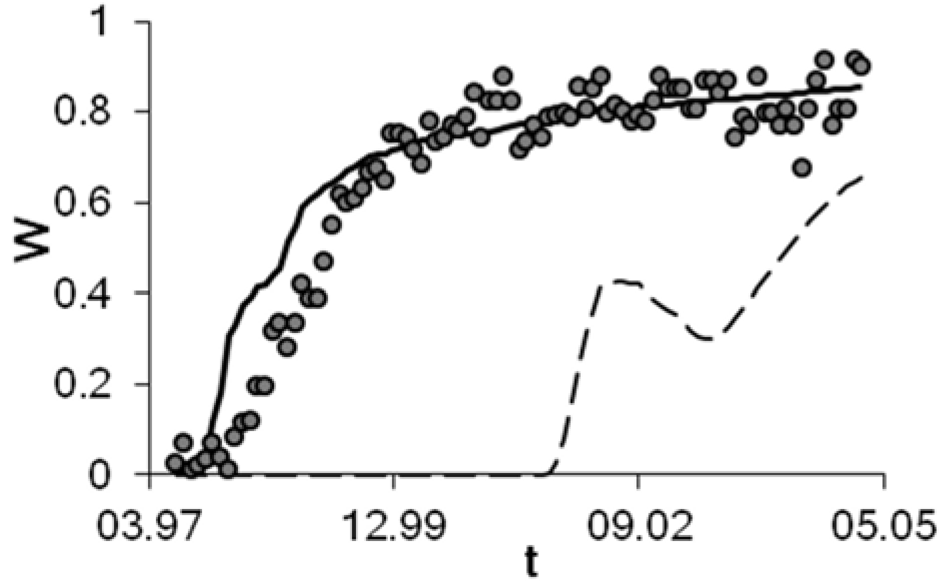


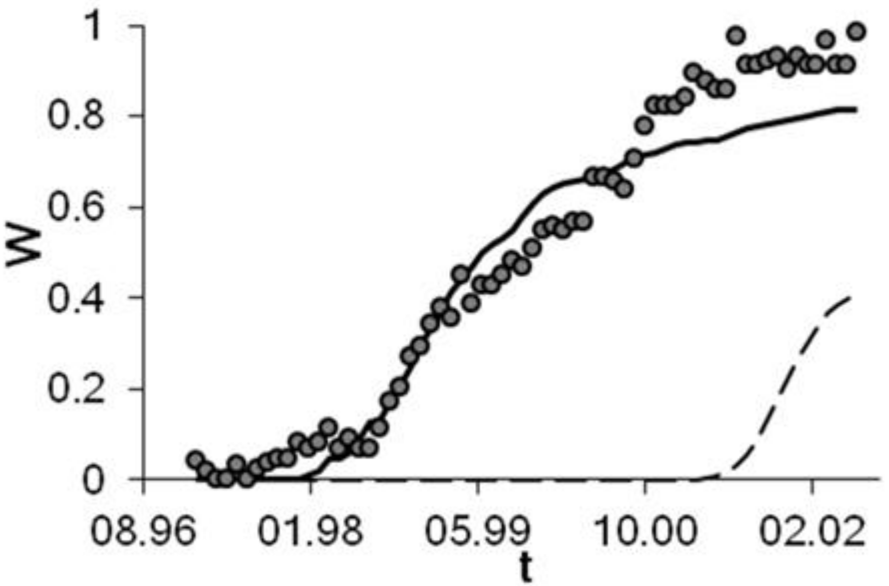

| $A_0$ | $A_1$ | $A_2$ | $B_0$ | $B_1$ | $B_2$ | $S_{min}$ |
|---|---|---|---|---|---|---|
| 0.54 | 4.48 | -4.14 | 3.18 | -9.56 | 9.73 | 0.23 |



# **Results.** Dynamic of watering on typical wells



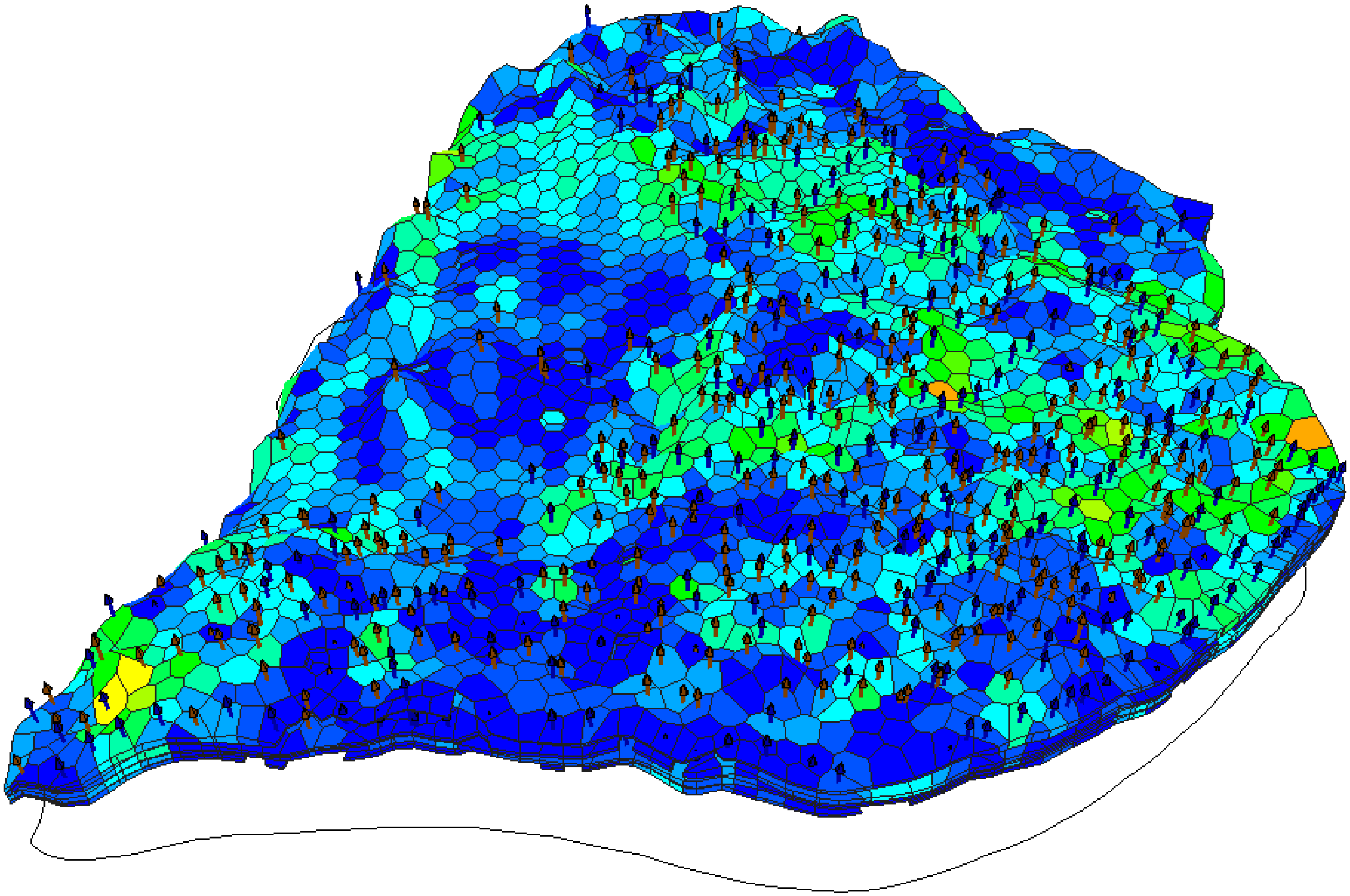

**markers** – actual values

**dashed line** – averaged reservoir with initial RP

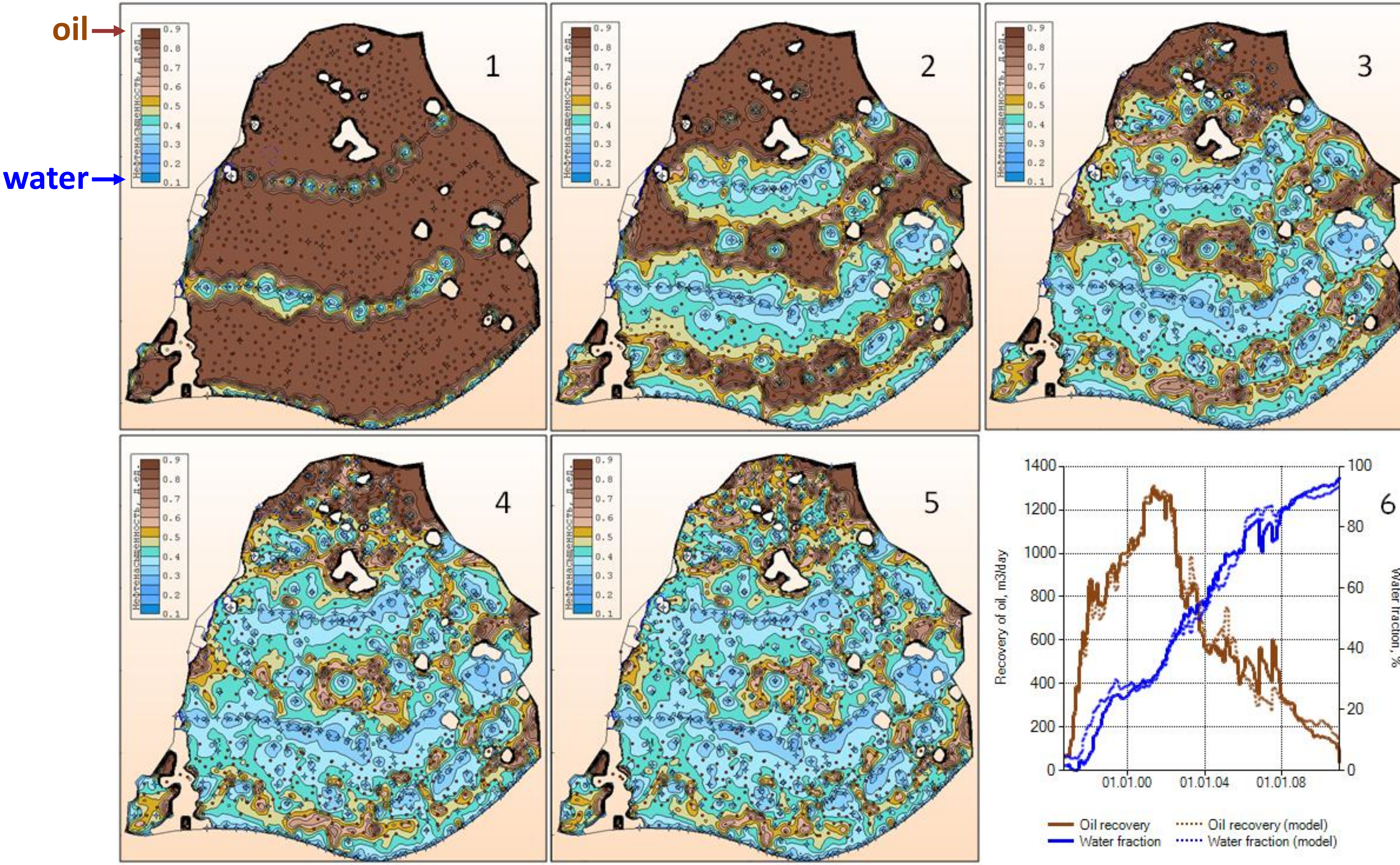
**solid line** – averaged reservoir with upscaled RP

# Simulation example for the area of Romashkinskoye field. Tatarstan

| Model parameters | | | |
|---|---|---|---|
| Wells number | 850 | Development duration (years) | 50 |
| 3D-superelements number | 7 567 | Calculation duration (minutes) | 4 |
| Average 2D-cell size (m) | 500 | Fact-model deviation (%) | 5 |



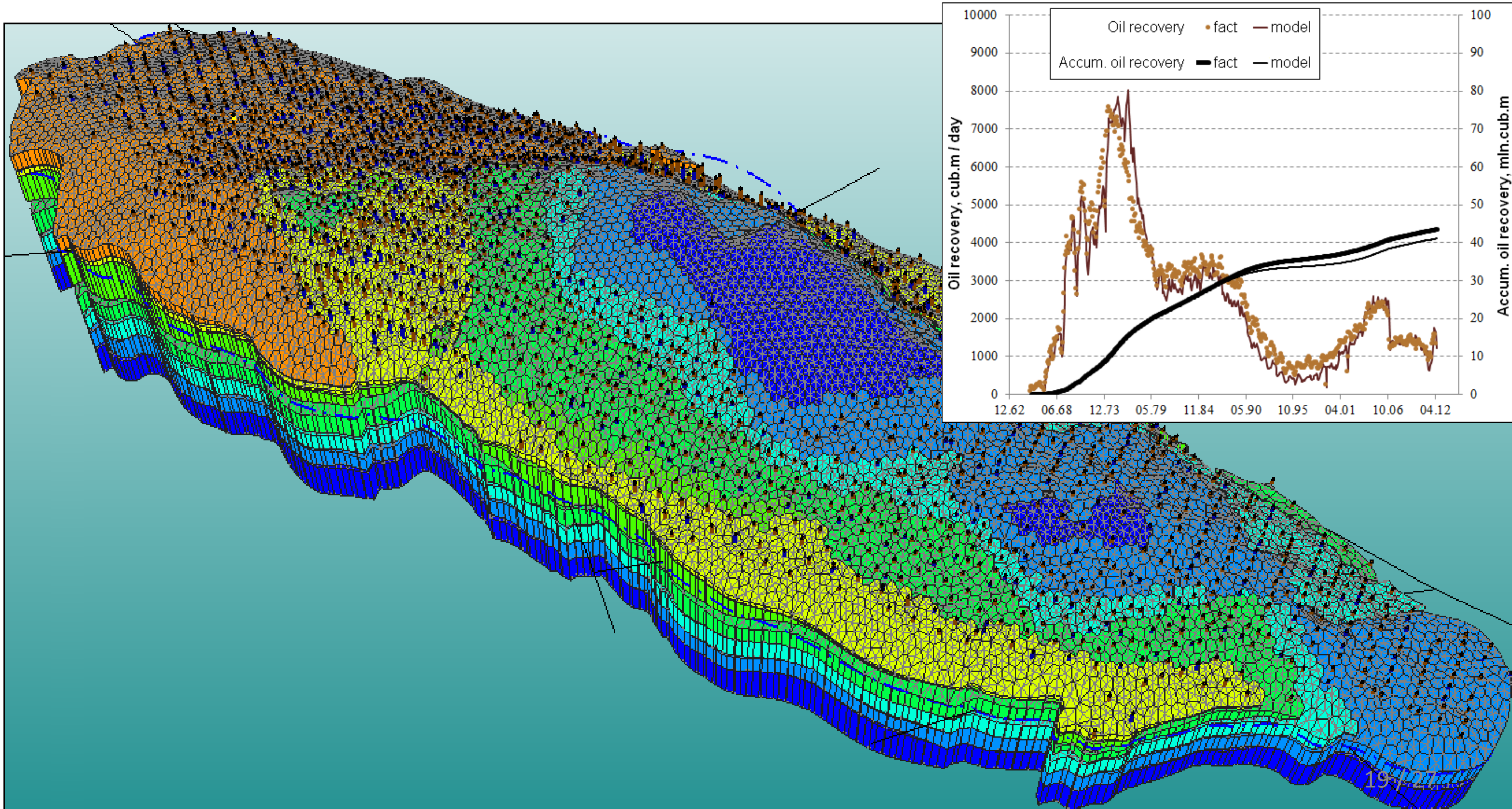
# Simulation example for the area of Romashkinskoye field. Tatarstan




Computational field of oil saturation (1-5) after each 10 years of development.
The graph of the calculated and real oil recovery and water fraction (6).

# Simulation example for an oil field of the Republic of Kazakhstan

| **Model parameters** | | 3D-superelements number | 41 731 |
|---|---|---|---|
| Wells number | 1 557 | Development duration (years) | 56 |
| Oil field area (sq. km) | 854 | Calculation duration (minutes) | 12.5 |
| Average 2D-cell size (m) | 350 | Fact-model deviation (%) | 6 |

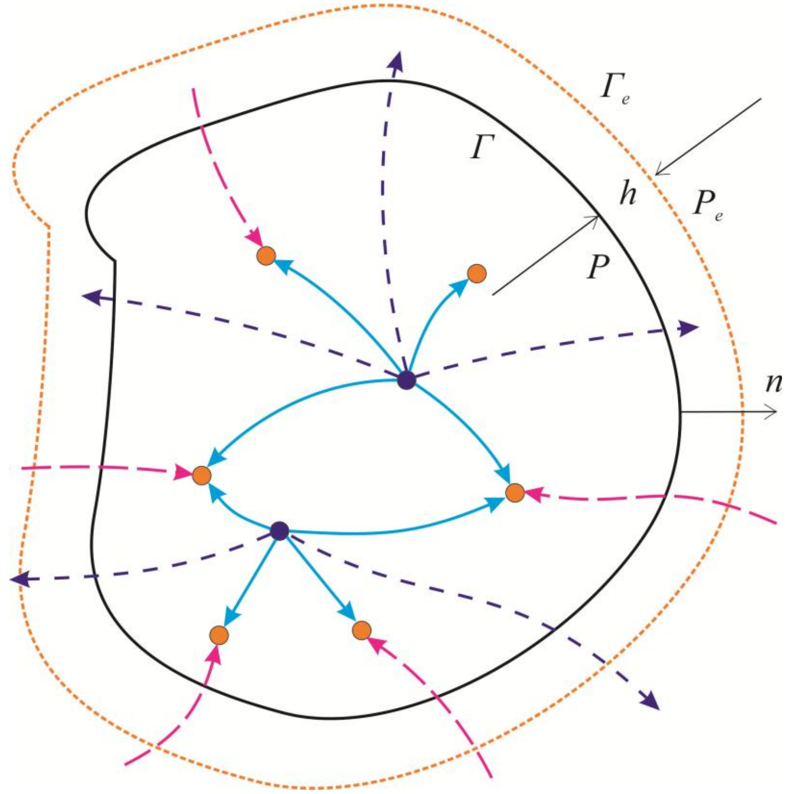


**ALGORITHM**

1. **Construction of a superelement solution for the whole history of the flooding of the oil deposit.**
2. **Unload local XYZ-model of the section.**
3. **Formulation of boundary conditions.**

$$\langle p \rangle \approx p, \quad \nabla \langle p \rangle \approx \nabla p$$

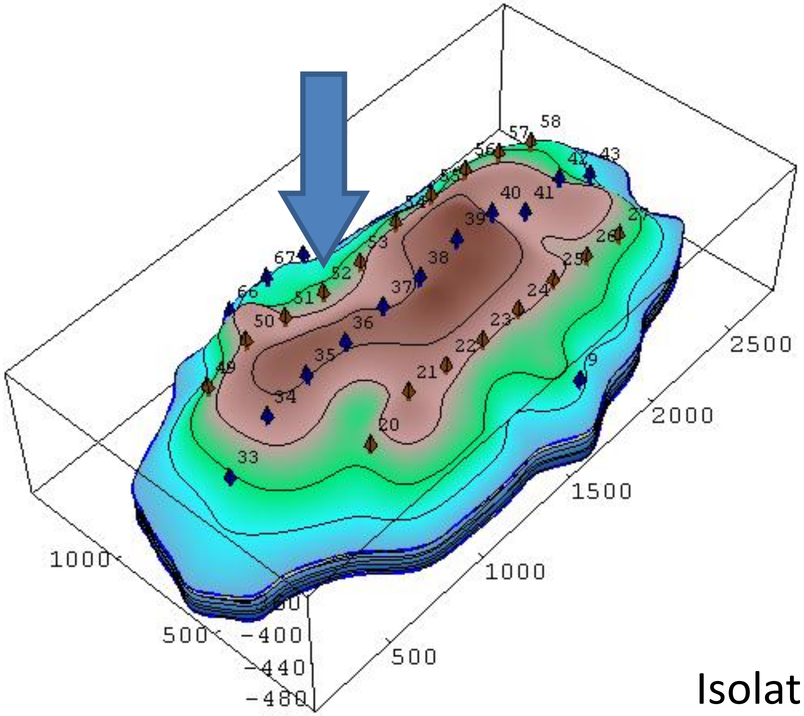
4. **Saturation downscaling and initial conditions formulation.**
5. **Solution of the local problem on detailed computational grid.**

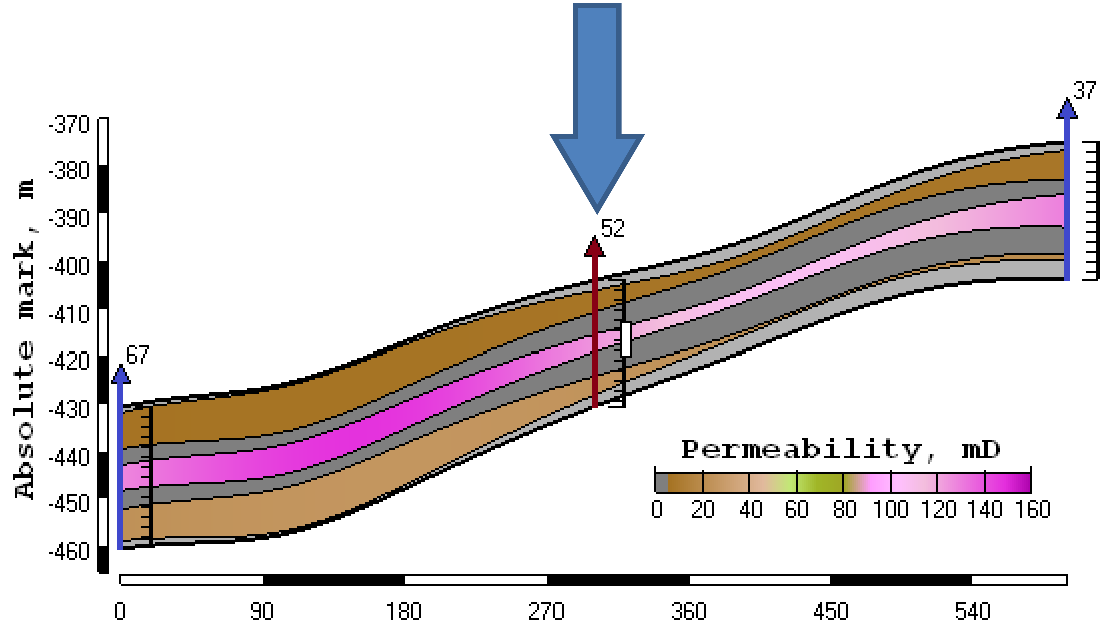



$$\mathbf{x} \in \Gamma: \quad \sigma \frac{\partial p}{\partial n} = -\alpha \left( p - P_e \right), \ \alpha \approx \frac{\sigma}{h}$$

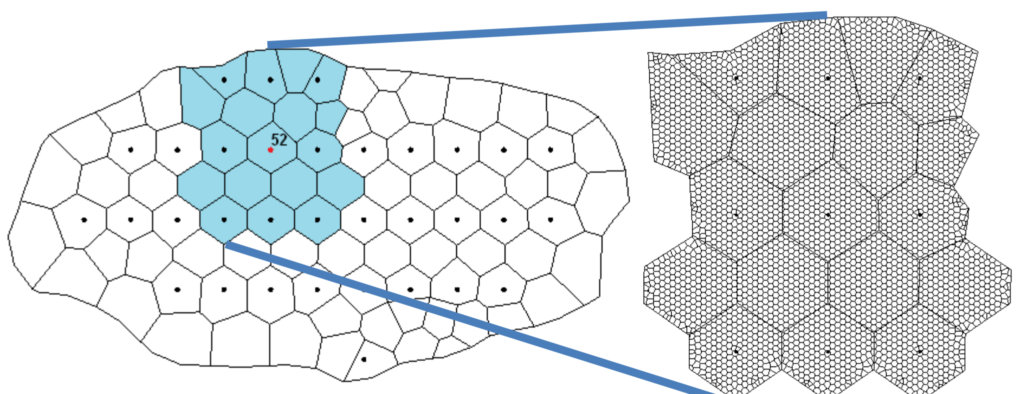
$$t = t_0, \ \mathbf{x} \in \Omega: \quad p = p^0(\mathbf{x}), \ s = s^0(\mathbf{x})$$

# Example of a local 3D detailing of the superelement solution




Isolation of the production well perforation interval
in the highly permeable watered seam




Covering of the reservoir
with coarse superelement grid

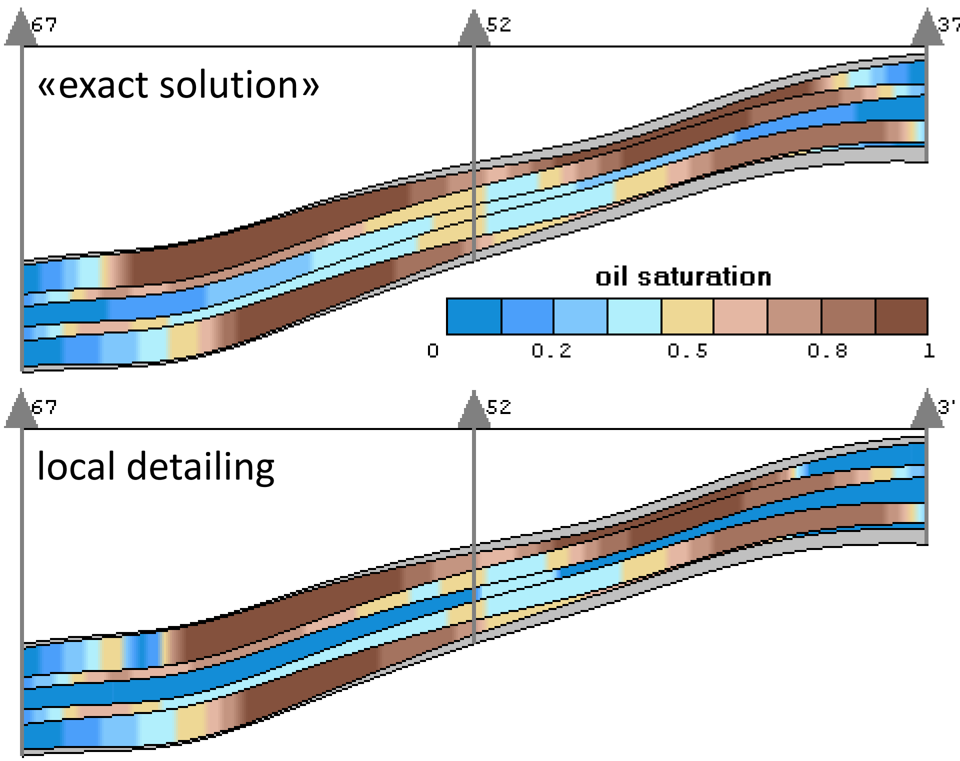
Covering of the sector
with a fine grid

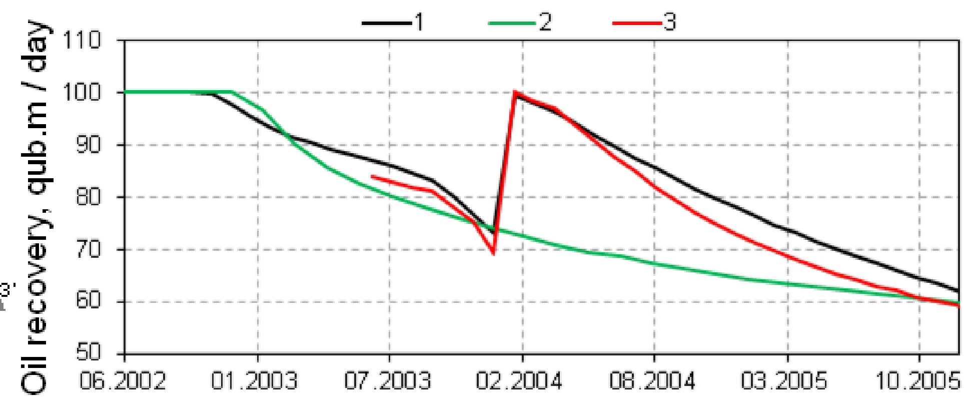
# Example of a local 3D detailing of the superelement solution

01.2000 start of the reservoir development

01.2004 filling the perforation interval

| Reservoir simulation on the coarse SE grid 01.2000 – 01.2006 **15 seconds** | Local detailing on the fine grid 07.2003 – 01.2006 **30 minutes** | «**Exact solution**» on the fine grid: 01.2000 – 01.2006 **4 hours** |
|---|---|---|




1 – «exact solution»

2 – SE model

3 – SE model + local detailing

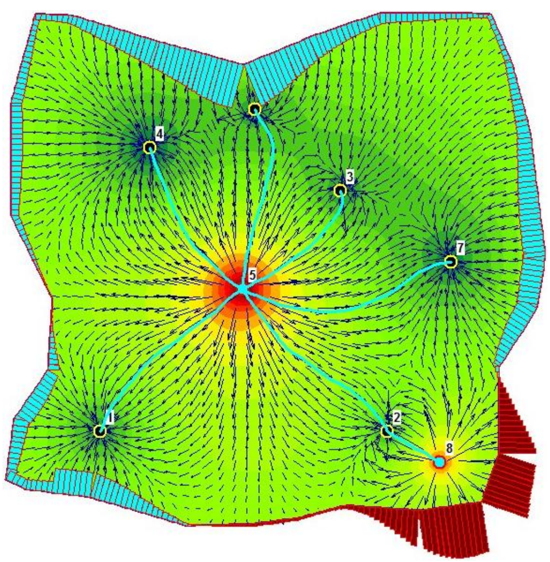
# **Local detailing.** Fixed streamtube model

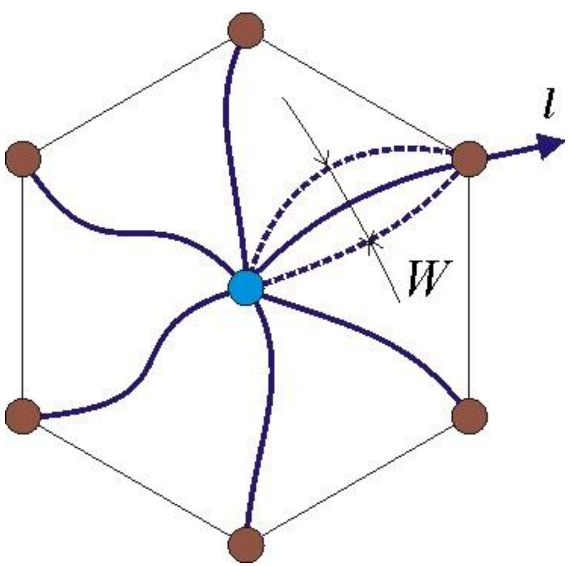
**ASSUMPTIONS**

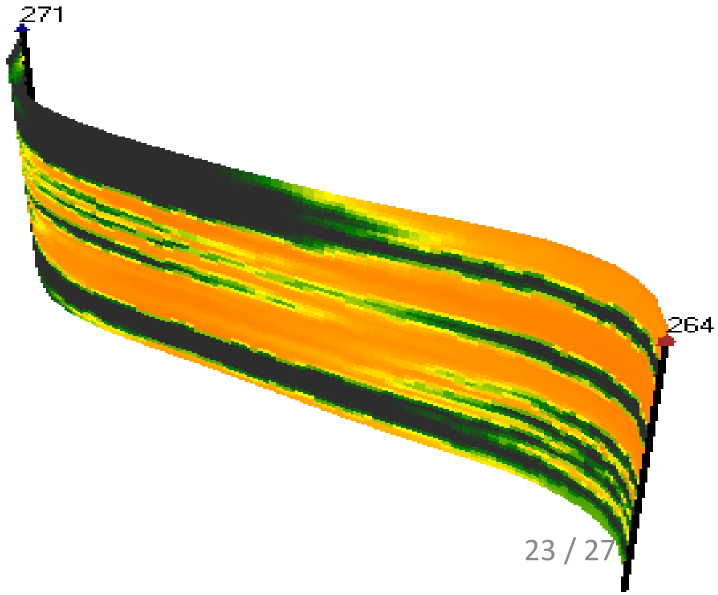
1. **"fast" events are modeled:**
   the boundary conditions
   and well operation modes are constant;
   the geometry of the streamtubes is fixed;
2. **wells are vertical, the distance between the wells does not exceed the scale of the lateral heterogeneity of the reservoir:**
   the XY-projections of the streamlines do not depend on Z; the streamtubes are bounded on the sides by vertical surfaces.

**ALGORITHM**

1. **3D superelement XYZ solution.**
2. **3D unload of XYZ-model of the sector.**
   Formulation of the boundary conditions.
3. **2D steady state XY problem.**
   Streamlines and streamtubes constructing.
   Interaction coefficients determination.
4. **2D problems of two-phase flow in the vertical lWz-section of each streamtube.**



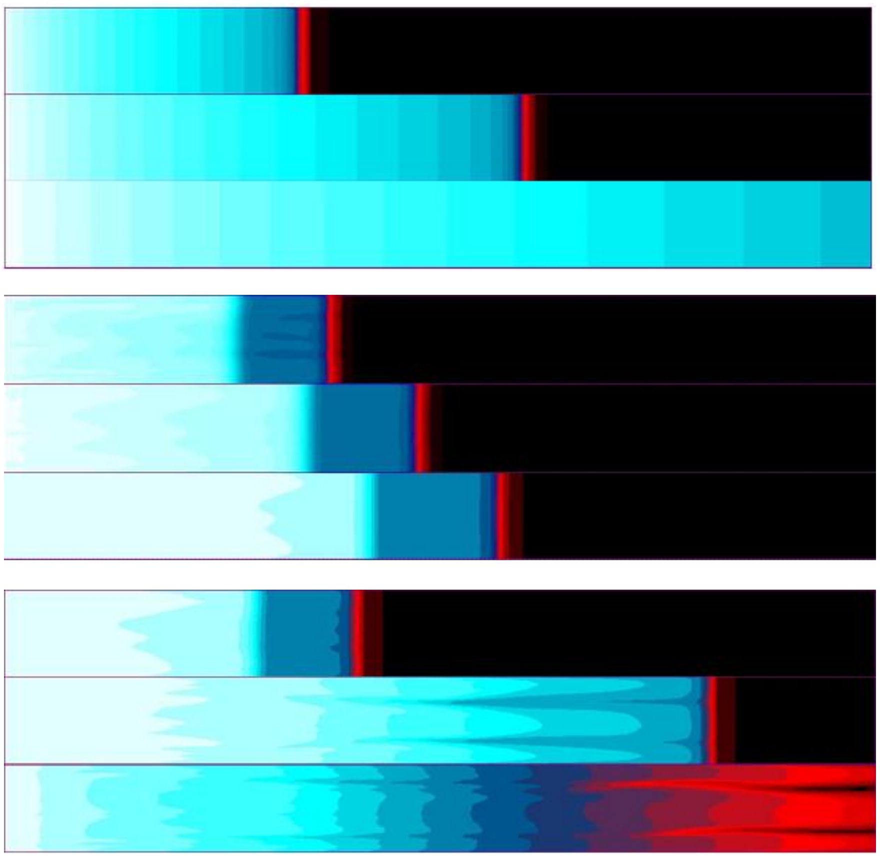


# Example of polymer flooding modeling using fixed streamtube model

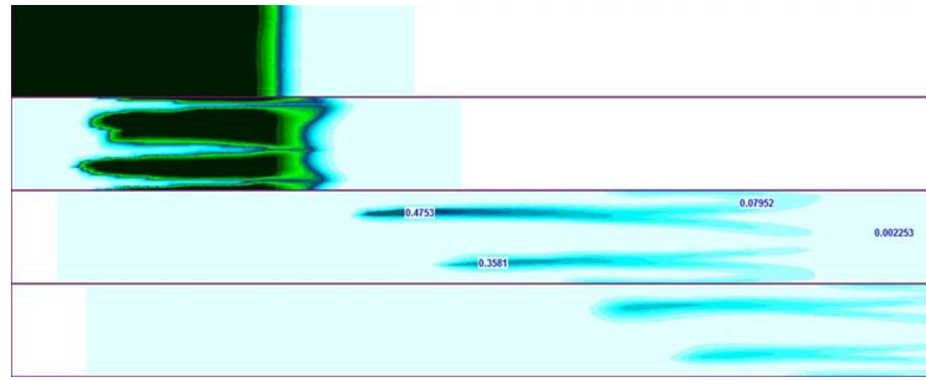
**Injection modes**:

1) without polymer
2) constant polymer concentration
3) with finite impulse of polymer

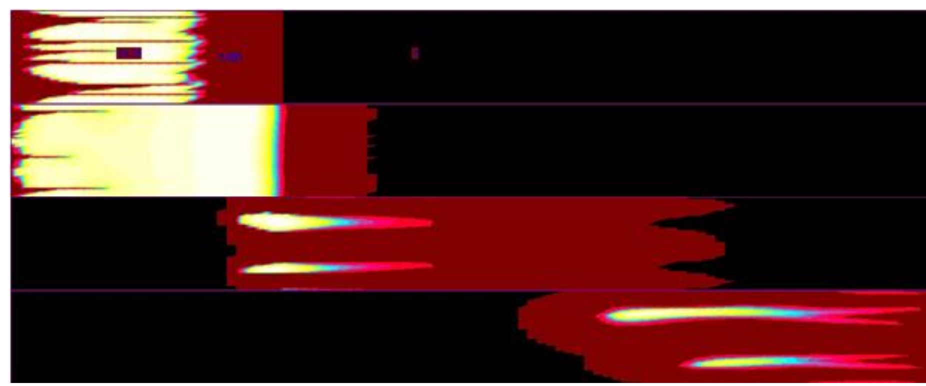


**Impulse mode of injection**

Destruction of the viscous polymer rim



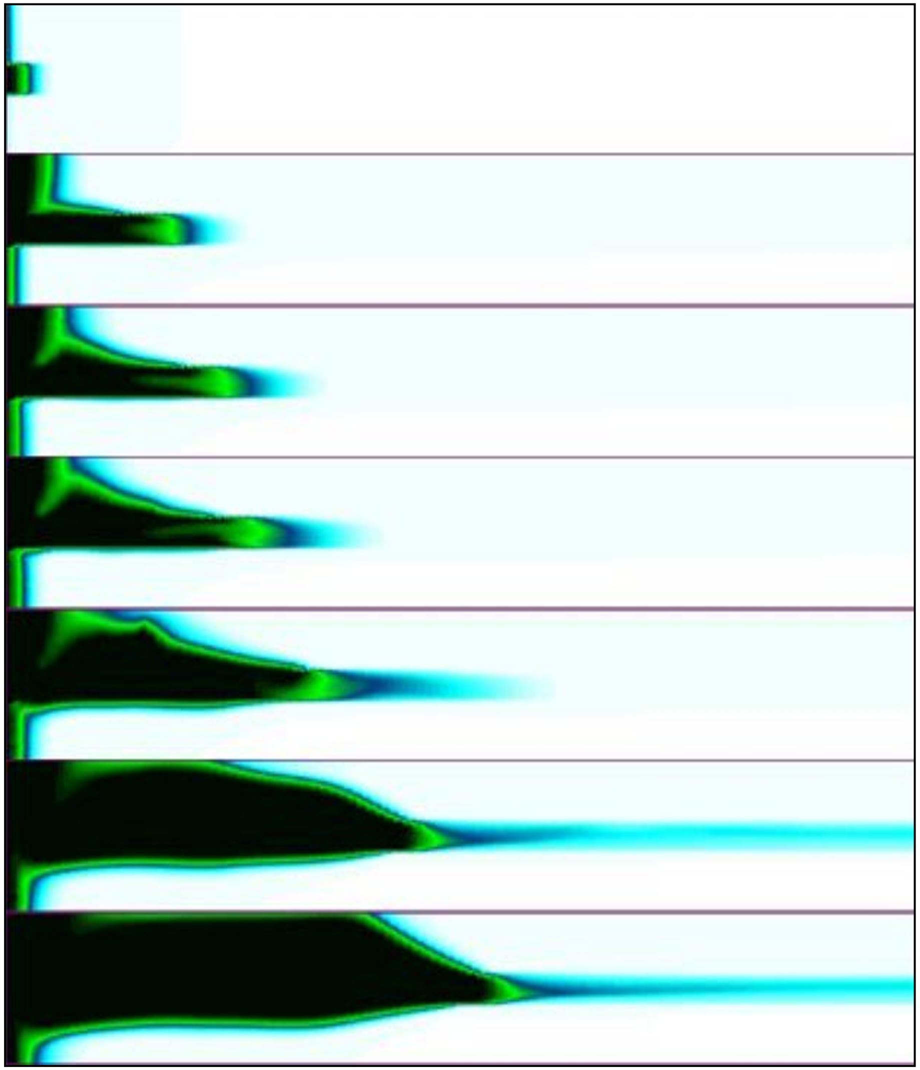

Polymer concentration fields
at different time moments



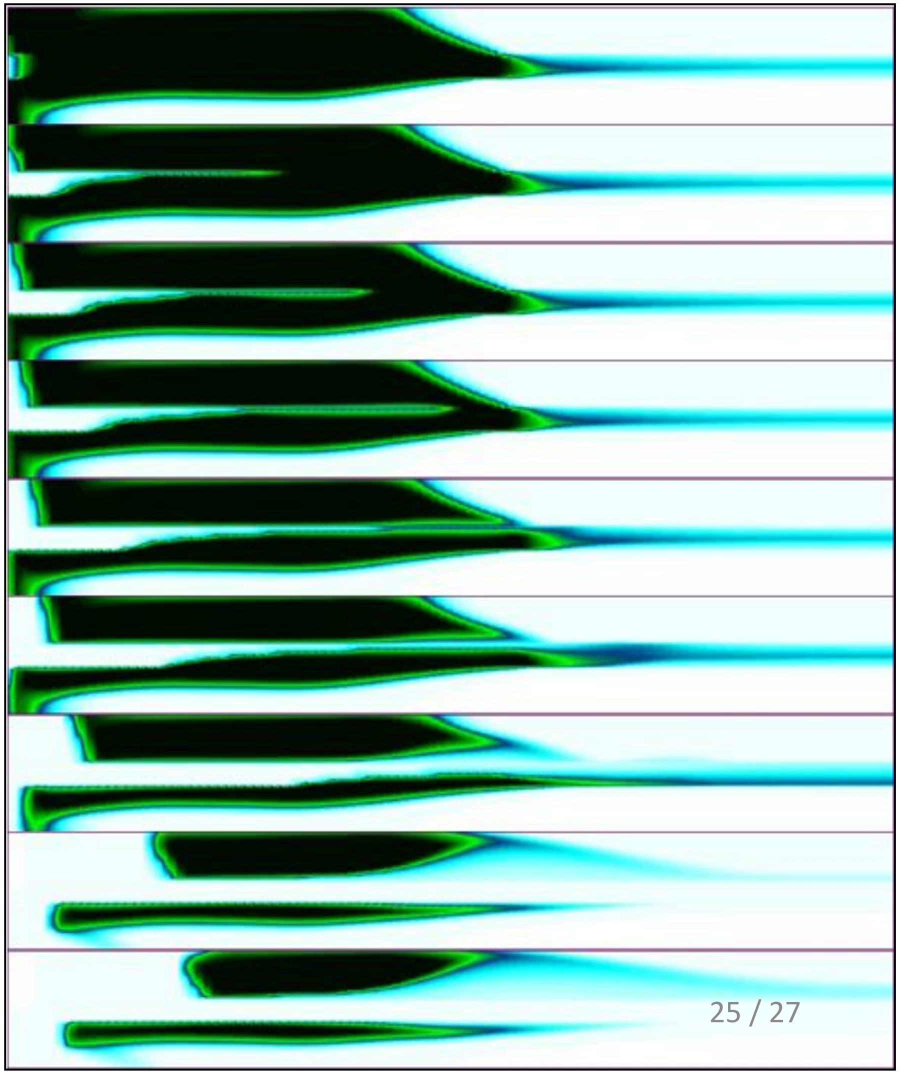
Effective viscosity fields
at different time moments

# Example of polymer flooding modeling using fixed streamtube model

Forming of high-viscosity polymer rim.
Impulse injection mode.



Destruction of the high-viscosity rim
during impulse polymer injection.
Polymer concentration fields
at different time moments.



# Geography of application of the developed models, methods and calculation programs

- **The Republic of Kazakhstan**
- **Tyumen region**
- **Surgut district, KhMAO**
- **Samara Region**
- **Republic of Tatarstan**
- **Moscow region**
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***Thank you for attention!***