ИВМ РАН

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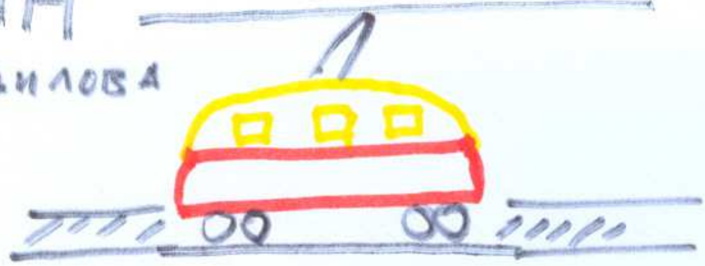

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Multicontinuum wave propagation in a laminated beam with contrasting stiffness and density of layers

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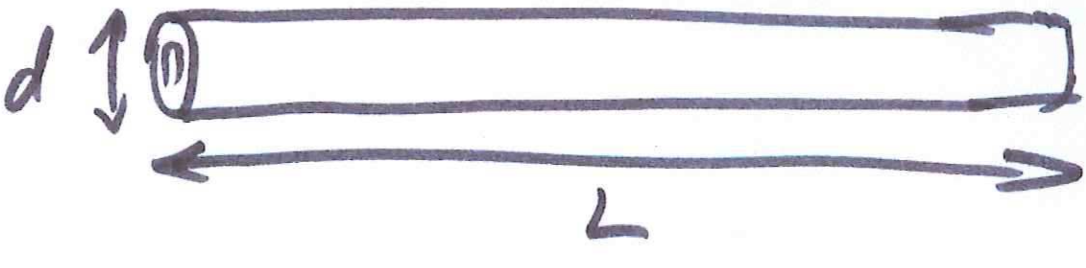
Outline

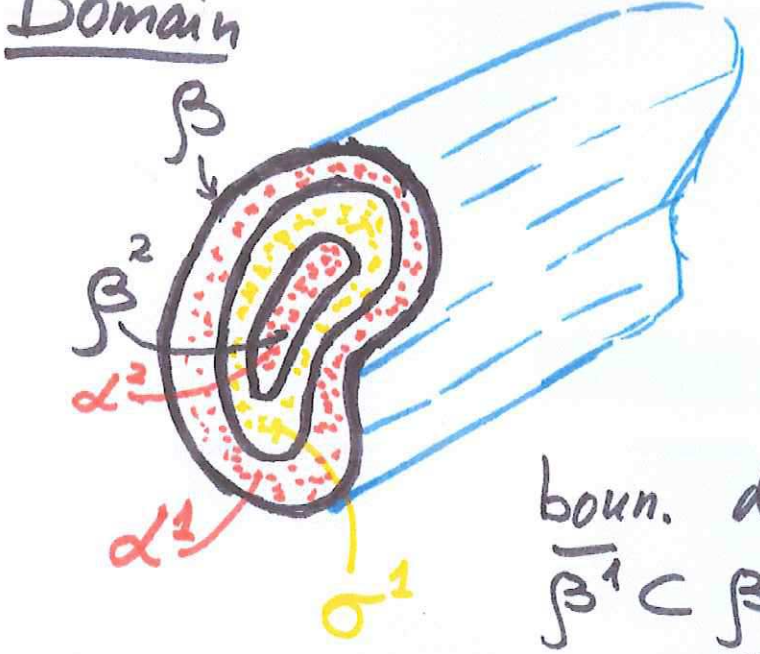
- Domain
- Equation
- Var. formulation
- Asymptotic behavior



$$\varepsilon = d/L \ll 1$$

$$\frac{\text{stiff}}{\text{weak}} = \omega \gg 1$$

1. Domain



Let β, β^l, $l = 1, \dots, L$ be Lipschitz $\mathbb{R}^{n-1}$ boun. domains: $\bar\beta^1 \subset \beta$, $\bar\beta^l \subset \beta^{l-1}$, $l \geq 2$.

$$\alpha^1 = \beta \setminus \overline{\beta^1}$$

$$\alpha^r = \beta^{2r} \setminus \overline{\beta^{2r+1}}$$

$$\sigma^r = \beta^{2r-1} \setminus \overline{\beta^{2r}} \qquad r \geq 1 \quad (\beta^l = \emptyset,\ l > L)$$

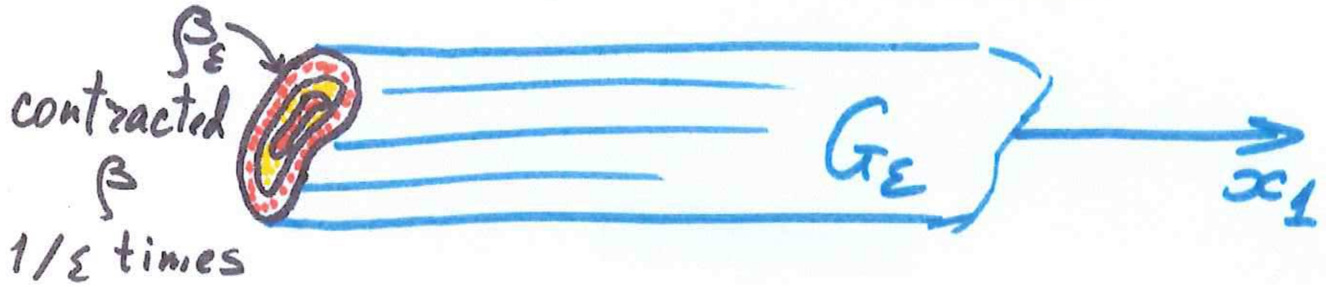
Consider constants $\rho, \rho^0, \rho^1, \dots, \rho^{L/2} > 0$ and $K, K^0, K^1, \dots, K^{L/2} > 0$; define

$$K_\omega(\xi') = \begin{cases} \underline{\omega} K^r & \text{if } \xi' \in \alpha^r \quad r = 0, \dots, L/2 \\ K & \text{if } \xi' \in \sigma^r \quad r = 1, \dots, L/2 \end{cases} \qquad \xi' = (\xi_2, \dots, \xi_n)$$

$$\rho_\omega(\xi') = \begin{cases} \underline{\omega}\, \rho^r & \text{in } \alpha^r \\ \rho & \text{in } \sigma^r \end{cases} \quad \text{and}$$

$$\underline{\omega} \to \infty$$

2. Wave eq. in the rod



$$\beta_\varepsilon := \left\{ x' \in \mathbb{R}^{n-1} \;\middle|\; \frac{x'}{\varepsilon} \in \beta \right\}$$

(same for α^2, σ^2)

$$G_\varepsilon = \mathbb{R} \times \beta_\varepsilon \; ; \; G_\varepsilon^1 = (0,1) \times \beta_\varepsilon$$

$$\begin{cases} \rho_\omega\left(\frac{x'}{\varepsilon}\right) \dfrac{\partial^2 u_{\omega,\varepsilon}}{\partial t^2} - \operatorname{div}\left(K_\omega\left(\frac{x'}{\varepsilon}\right) \nabla u_{\omega,\varepsilon}\right) = f_\omega(x_1,t) \\ \qquad\qquad x \in G_\varepsilon, \; t > 0 \qquad (1) \\ K_\omega\left(\frac{x'}{\varepsilon}\right) \dfrac{\partial u_{\omega\varepsilon}}{\partial n} = 0, \quad x \in \partial G_\varepsilon \qquad (2) \\ u_{\omega,\varepsilon}\Big|_{t=0} = 0, \quad \dfrac{\partial u_{\omega,\varepsilon}}{\partial t}\Big|_{t=0} = 0 \qquad (3) \end{cases}$$

where $f_\omega(x_1,t) = \omega f(x_1,t) \in C^\infty_{per}(\mathbb{R} \times [0,T])$, 1-periodic in x_1, $f(x_1,t) = 0$ for $t \le \tau$.

On $\partial\beta_\varepsilon^l$? $[u_{\omega\varepsilon}] = 0$, $\left[K_\omega \dfrac{\partial u_{\omega\varepsilon}}{\partial n}\right] = 0$. (4)

3. Variational formulation

Let $H^1_{per}(G_\varepsilon)$ be the space of functions $v \in H^1((-R,R)\times \beta_\varepsilon)$ $\forall R$, v 1-periodic in x_1.

Def. $u_{\omega\varepsilon}$ is a solution to (1) – (4) if $u_{\omega\varepsilon} \in H^2(0,T; H^1_{per}(G_\varepsilon))$, satisfies (3) and $\forall\, v \in H^1_{per}(G_\varepsilon)$, $\forall\, t \in (0,T)$,

$$\int_{G^1_\varepsilon} \left(\rho_\omega \frac{\partial^2 u_{\omega\varepsilon}}{\partial t^2} v + K_\omega \nabla u_{\omega\varepsilon} \cdot \nabla v \right) dx = \int_{G^1_\varepsilon} f_\omega v \, dx . \qquad (5)$$

Th. $\exists !$ sol.

Let $\|v\|_V^2 = \frac{1}{2} \sup_{t\in[0,T]} \int_{G^1_\varepsilon} \rho_\omega \left(\frac{\partial v}{\partial t}\right)^2 + K_\omega (\nabla v)^2 dx$

then $\|u_{\omega\varepsilon}\|_V \le C_T \|f_\omega\|_{L^2(G^1_\varepsilon \times (0,T))}$

$\le C \|f_\omega\|_{L^1(0;T; L^2(G^1_\varepsilon))}$

4. Convergences.

$u_{\omega\varepsilon}$ L-converges to $u^a_{\omega\varepsilon}$ $\left(u_{\omega\varepsilon} \xrightarrow{L} u^a_{\omega\varepsilon}\right)$

$$\| u_{\omega\varepsilon} - u^a_{\omega\varepsilon} \|_{L^2} / \sqrt{\operatorname{mes} G^1_\varepsilon} \to 0 .$$

(G.P. Homog. of lattice-like domains: L-convergence College de France sem. V. XIII 259-280, 1998).

$\boxed{\text{Case 1.} \quad \varepsilon^2\omega \to 0.}$ $\quad u_{\omega\varepsilon} \xrightarrow{L} v_0$:

$$\begin{cases} \langle \rho \rangle \dfrac{\partial^2 v_0}{\partial t^2} - \langle K \rangle \dfrac{\partial^2 v_0}{\partial x_1^2} = f(x_1, t) \operatorname{mes} \rho , \\ \qquad\qquad x_1 \in \mathbb{R}, \; t > 0, \\ v_0 \text{ 1-per. in } x_1 \\ v_0 \big|_{t=0} = 0 ; \; \dfrac{\partial v_0}{\partial t} \Big|_{t=0} = 0 \end{cases}$$

Here $\langle F(\xi') \rangle = \sum\limits_{r=1}^{L/2} \langle K_r \rangle_r$,

$\langle F(\xi') \rangle_r = \int\limits_{\alpha_r} F(\xi') d\xi'$ (for $F = \text{const}$,

$\langle F \rangle_r = F \operatorname{mes} \alpha_r$).

Proof: as in G.P. Homog. of periodic structures with well conducting inhomogeneities. Vestnik Moscow Univ., Appl. Math., 1980, 3, 4-11.

Case 2. $\varepsilon^2\omega \to +\infty$ or $\varepsilon^2\omega = \varkappa = cst$

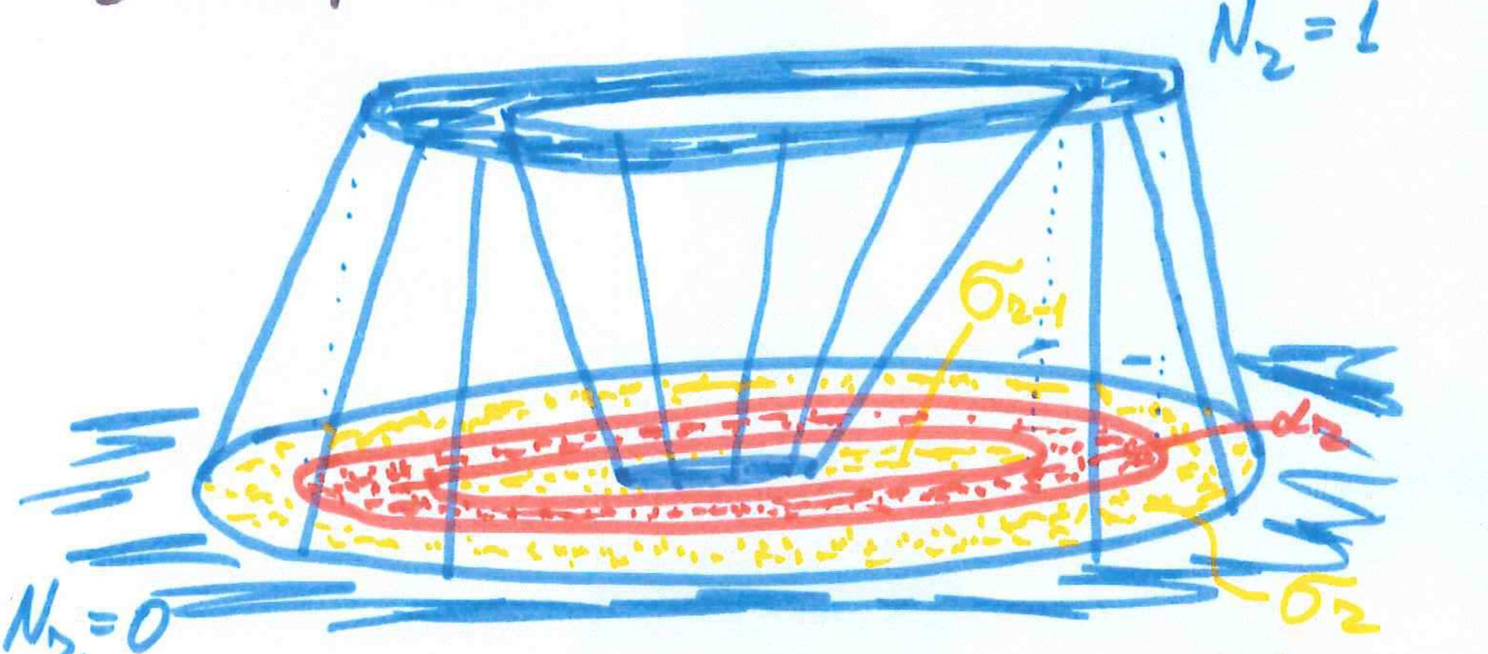
$$u_{\omega\varepsilon} \xrightarrow{L} u^a_{\omega\varepsilon} \quad \text{where}$$

$$u^a_{\omega\varepsilon}(x,t) = \sum_{r=0}^{L/2} N_r\left(\frac{x'}{\varepsilon}\right) v_r(x_1,t) + \varepsilon^2\omega\, M_0\left(\frac{x'}{\varepsilon}\right) f(x_1,t)$$

$N_r(\xi')$ solution to
$$\begin{cases} \Delta N_r = 0, & \xi' \in \sigma_r \cup \sigma_{r-1} \\ N_r = 1, & \xi' \in (\partial\sigma_r \cup \partial\sigma_{r-1}) \cap \partial d_r \\ N_r = 0, & \xi' \in (\partial\sigma_r \cup \partial\sigma_{r-1}) \setminus \partial d_r \end{cases}$$

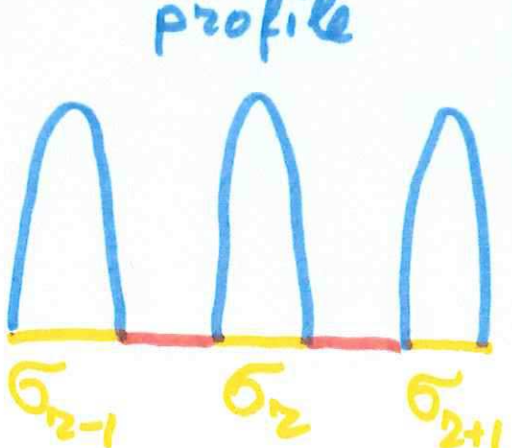
extended $N_r = 1$ in d_r

$N_r = 0$ out of $\overline{\sigma_r \cup d_r \cup \sigma_{r-1}}$



$$M_0(\xi') : \begin{cases} -K\,\Delta M_0 = 1, & \xi' \in \sigma_r \\ M_0|_{\partial\sigma_r} = 0, & r = 1, \dots \end{cases}$$

extended 0 in $\bigcup_{r \geq 1} d_r$



Case 2a: $\varepsilon^2\omega \to +\infty$.

$$\begin{cases} \langle \rho^2 \rangle_z \dfrac{\partial^2 v_z}{\partial t^2} - \langle K^2 \rangle_z \dfrac{\partial^2 v_z}{\partial x_1^2} = \langle f \rangle_z, \\ \qquad\qquad x_1 \in \mathbb{R}, t > 0 \\ v_z \text{ 1-per in } x_1, \\ v_z\big|_{t=0} = 0; \ \dfrac{\partial v_z}{\partial t}\Big|_{t=0} = 0 . \end{cases}$$

Case 2b: $\varepsilon^2\omega = æ$

$$\begin{cases} \langle \rho^2 \rangle_z \dfrac{\partial^2 v_z}{\partial t^2} - \langle K^2 \rangle_z \dfrac{\partial^2 v_z}{\partial x_1^2} + \\ + æ^{-1}\left(\mathcal{D}_z (v_z - v_{z+1}) - \mathcal{D}_{z-1}(v_{z-1} - v_z)\right) = \langle f \rangle_z \\ \qquad\qquad x_1 \in \mathbb{R}, t > 0, \\ v_z \text{ 1-per in } x_1, \\ v_z\big|_{t=0} = 0; \ \dfrac{\partial v_z}{\partial t}\Big|_{t=0} = 0, \end{cases}$$

$$\mathcal{D}_z = -K \int_{\partial\sigma_z \cap \partial\alpha_z} \frac{\partial N_z}{\partial n} \, ds, \quad n \text{ outer w.r.t. } \alpha_z$$

It is a multicomponent homogenized model : [L/2] independent rods for each x_1, t.
[L/2] wave velocities (2a)
or weakly dependent (2b)

Proof: method of multicomponent homog.

G.P. On the scale effect in spacially reinforced composites. Proc. of 2nd USSR Conf. on Strength, Rigidity and Technol. of Composite Materials V.3, 22-24, Yerevan, 1984

G.P. Homogenization of processes in strongly non-homog. media. USSR Doklady 298, 1:76-79 1988

G.P. Multicomponent homogenization of processes in strongly non-homog. structures Math. USSR Sb. 181, 1, 134-142, 1990

(Spacial reinforcement)

Complete asymptotic expansion is constructed and justified in the norm V.

Some related topics

1) Weakly connected domains
E. Ya. Khruslov: in Theory of Operators in Functional Spaces and its Applications (Ed. V. A. Marchenko) Naukova Dumka Kiev, 1981, 129–174

2) Double porosity model (T. Arbogast, 1990)

THANK YOU FOR ATTENTION!