Two-Level Approach for Modeling Blood Flow in Liver Lobule

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Prerequisites

- In work A. Bonfiglio e.t.c. was considered numerical modeling blood flow in liver lobule.
- In work M. Dufresne was considered structure of liver lobule. As result we can conclude that lobule have double porosity structure

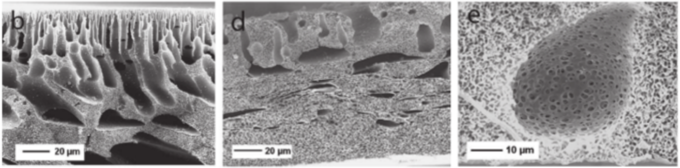
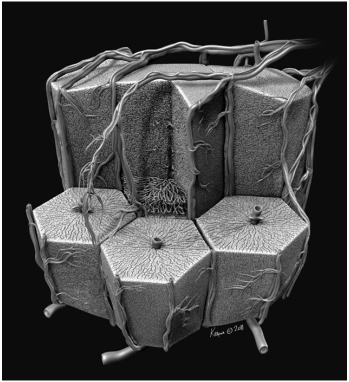


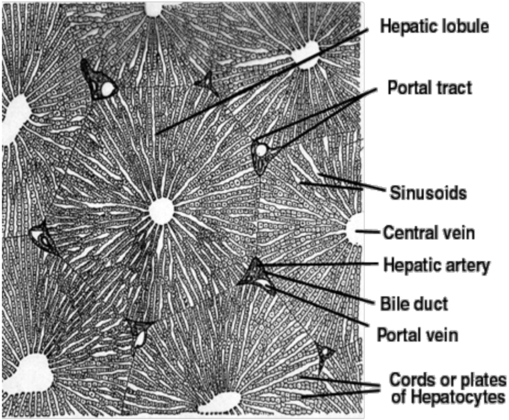
Figure: The pictures on the electron microscope

- Liver size 30 cm and weight 1,5 kilogram.
- Functions: Protective, hematopoietic, energy storage.
- The liver is a natural filter of our body.

Lobule structure



a



b

Figure: Lobule: a — 3D view, b — lobule geometry

Blood flow based on double porosity approach

Popular mathematical model for describing the flow in cavity porous media has been proposed by Barenblatt, Zheltov, Kochina. This is the classic method used for oil production.

$$\boldsymbol{v}_\alpha = -\frac{k_\alpha}{\mu}\nabla p_\alpha,$$

$$\frac{\partial c_\alpha \rho}{\partial t} + \operatorname{div} \rho \boldsymbol{v}_\alpha = q_\alpha,$$

$$q^\alpha = q^\alpha(p^\alpha, \rho, \mu), \quad m^\alpha = m(p^\alpha), \quad \alpha = 1, 2,$$

here $\boldsymbol{v}_\alpha$ — velocity vector, k_α — permeability components, c_α — porosity coefficients.

Mathematical model adaptation

We consider blood as a weakly compressible Newtonian fluid ($\rho \approx \texttt{const}$).

$$c_1 \frac{\partial p_1}{\partial t} - \mathrm{div}\left(K_1 \nabla p_1\right) + r\left(p_1 - p_2\right) = f_1,$$

$$c_2 \frac{\partial p_2}{\partial t} - \mathrm{div}\left(K_2 \nabla p_2\right) - r\left(p_1 - p_2\right) = f_2,$$

here $\boldsymbol{v}_\alpha(\boldsymbol{x})$, $p_\alpha(\boldsymbol{x})$ velocities and pressure , $r(p_1 - p_2)$ express exchange between two continua.

Lobule scheme

$$p_\alpha = p_\alpha^p, \quad \boldsymbol{x} \in \Gamma_p,$$

$$p_\alpha = p_\alpha^v, \quad \boldsymbol{x} \in \Gamma_v,$$

$$\nabla p_\alpha \cdot \boldsymbol{n} = 0, \quad \boldsymbol{x} \in \Gamma_b.$$

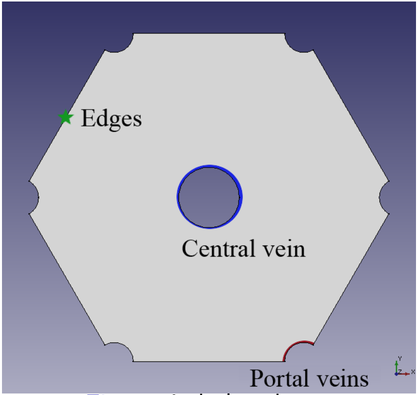


Figure: Lobule scheme

Dimensionless problem

$$p_\alpha^* = \frac{p_\alpha - p_\alpha^v}{p_\alpha^p - p_\alpha^v}, \quad \boldsymbol{x}^* = \frac{\boldsymbol{x}}{L},$$

$$p_\alpha^* = 1, \quad \boldsymbol{x} \in \Gamma_p,$$

$$p_\alpha^* = 0, \quad \boldsymbol{x} \in \Gamma_v,$$

$$\nabla p_\alpha^* \cdot \boldsymbol{n} = 0, \quad \boldsymbol{x} \in \Gamma_b,$$

$$p_\alpha^* = 0, \quad t = 0.$$

Computational basis

- Numerical realization of problem based on the FEM ($\triangle$ mesh)
- Lagrangian finite elements of the first degree

$$V = v \in H^1(\Omega) : \quad v(\boldsymbol{x}) = c, \quad \boldsymbol{x} \in \Gamma_D,$$

$$c_1(\frac{\partial p_1}{\partial t}, v_1) + a_1(p_1, v_1) + r(p_1 - p_2, v_1) = (f_1, v_1), \quad \forall\, v_1 \in V,$$

$$c_2(\frac{\partial p_2}{\partial t}, v_1) + a_2(p_2, v_2) - r(p_1 - p_2, v_2) = (f_2, v_2), \quad \forall\, v_2 \in V,$$

$$c_\alpha(p, v) = c_\alpha \int_\Omega p\, v d\boldsymbol{x}, \quad a_\alpha(p, v) = \int_\Omega (K \nabla p, \nabla v)\, d\boldsymbol{x},$$

Estimate and time scheme

$$\|p_1\|^2_{c_1} + \|p_2\|^2_{c_2} \leq \|p_1^0\|^2_{c_1} + \|p_2^0\|^2_{c_2} + \frac{1}{2}\int_0^T \|f_1(t)\|^2_{a_1} + \|f_2(t)\|^2_{a_2}\, dt,$$

Finite dimensional V_h:

$$\|p_{1,h}\|^2_{C_1} + \|p_{2,h}\|^2_{C_2} \leq \|p_{1,h}^0\|^2_{C_1} + \|p_{2,h}^0\|^2_{C_2} + \frac{1}{2}\int_0^T \|f_{1,h}(t)\|^2_{A_1} + \|f_{2,h}(t)\|^2_{A_2}\, dt$$

- Fully implicit scheme;
- Splitting through previous layer values in exchange.

Triangulated mesh

The results of numerical calculations are performed on a sequence of refined grids.

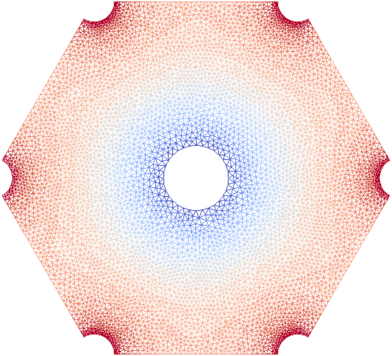


Figure: Mesh: 6117 vertices, 11874 elements

Numerical results

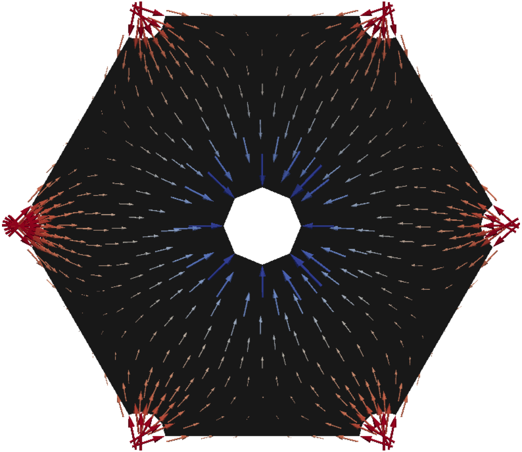
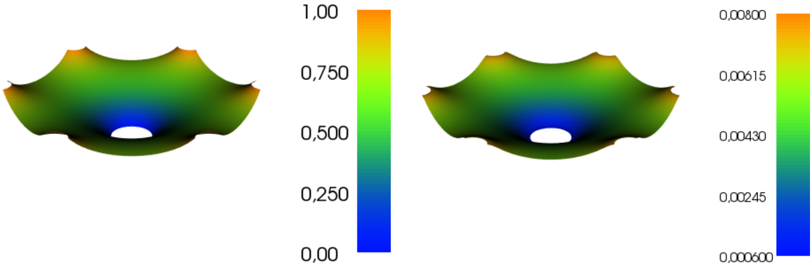


Figure: Velocity field

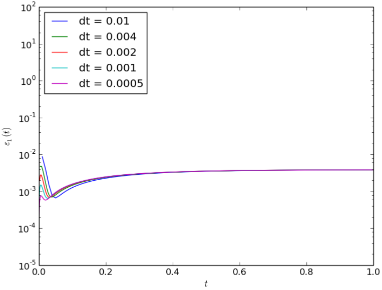
Pressure distribution



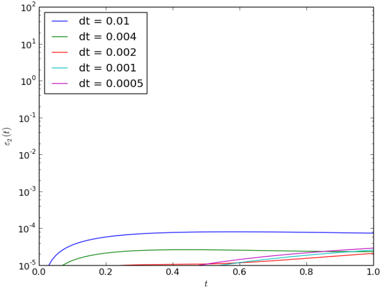
a b

Figure: Results in: a — sinusoids, b — pores

Methodical calculations



a



b

Figure: Error norm L_2: a — capillars, b — pores

Anisotropy tensor of flow

Let's consider general case with anisotropy of sinusoids in lobule.
Introduce anisotropy tensor in polar c.s.

$$K_1 = \begin{pmatrix} K_r & 0 \\ 0 & K_\varphi \end{pmatrix}$$

Same tensor in Cartesian coordinates

$$K_1 = \frac{1}{x^2 + y^2} \begin{pmatrix} K_r x^2 + K_\varphi y^2 & xy(K_r - K_\varphi) \\ xy(K_r - K_\varphi) & K_r y^2 + K_\varphi x^2 \end{pmatrix}$$

System with anisotropy

$$c_1 \frac{\partial p_1}{\partial t} - \operatorname{div} K_1 \operatorname{grad} p_1 + r\,(p_1 - p_2) = 0,$$

$$c_2 \frac{\partial p_2}{\partial t} - d \operatorname{div} \operatorname{grad} p_2 - r\,(p_1 - p_2) = 0.$$

with next physical parameters
$c_1 = 0.2,\ c_2 = 0.8,\ d = 0.01,\ r = 0.1,\ K_r = 2,\ K_\varphi = 0.5$
and numerical parameters
$T = 2,\ \tau = 0.01$, mesh consist of 17702 elements.
Also consider similar Initial and BCs.

Anisotropic - Isotropic

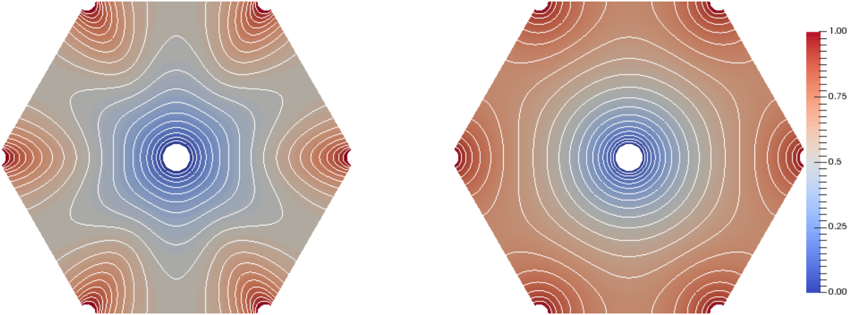
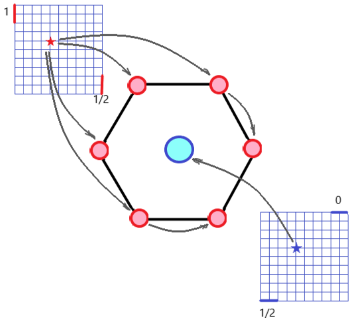


Figure: Pressure in sinusoids at $t = 2$: Left - Anisotropic, Right - Isotropic

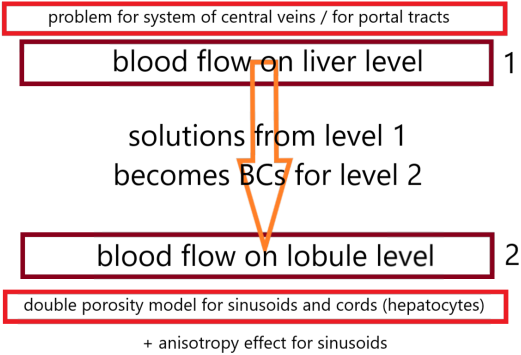
Basic assumptions

- Liver contains ≈ 10000 lobules ($D_L = 30$ cm, $D_l = 2$ mm)
- Lobule contains hundreds of sinusoids
- Size of lobule small enough compared to size of liver (part)
- Side of hex much smaller than height of cylinder
- Capacity of lobule small enough

Lobule structure



a



b

Figure: Two level: a — picture, b — scheme

Liver level problems

Model can be reduced from original applying $r = 0$.

$$c_1^u \frac{\partial p_1^u}{\partial t} - \operatorname{div}\left(K_1^u \nabla p_1^u\right) = 0,$$

$$c_2^u \frac{\partial p_2^u}{\partial t} - \operatorname{div}\left(K_2^u \nabla p_2^u\right) = 0,$$

Add boundary $p_1^u = 1 \quad$ at $x = 0, \quad 0.8 \le y \le 1$,
$p_1^u = 0.5 \quad$ at $x = 1, \quad 0 \le y \le 0.2$,
$p_2^u = 0 \quad$ at $y = 1, \quad 0.8 \le x \le 1$,
$p_2^u = 0.5 \quad$ at $y = 0, \quad 0 \le x \le 0.2$,
and initial conditions
$p_1^u(0) = 0$, $p_2^u(0) = 0$.

Behavior of Level 1

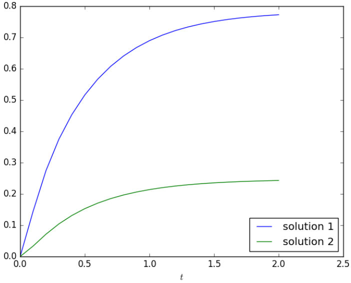


Figure: Boundary conditions source

Solution of two-level approach with anisotropy $t = 0.5$

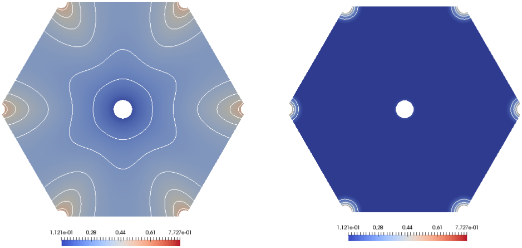


Figure: Pressure left in sinusoids, right in sinusoidal space

Solution of two-level approach with anisotropy $t = 1$

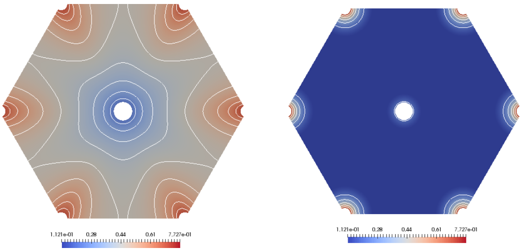


Figure: Pressure left in sinusoids, right in sinusoidal space

Solution of two-level approach with anisotropy $t = 1.5$

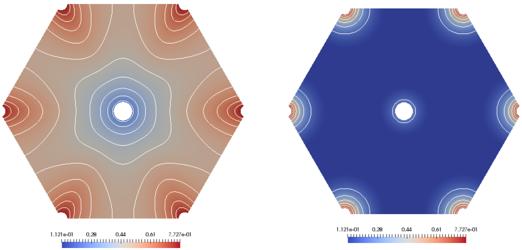


Figure: Pressure left in sinusoids, right in sinusoidal space

Solution of two-level approach with anisotropy $t = 2$

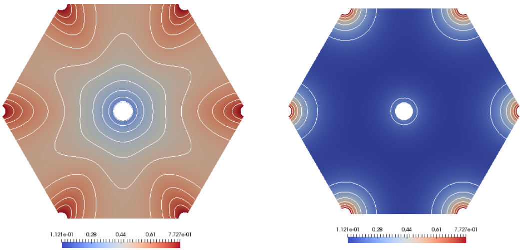


Figure: Pressure left in sinusoids, right in sinusoidal space

Comparison with Bonfiglio article results

1. realization: FEniCS / Comsol;
2. blood as: weakly compressible / incompressible;
3. double porosity (sinusoids) / without double porosity (averaged);
4. proposed two-level approach.

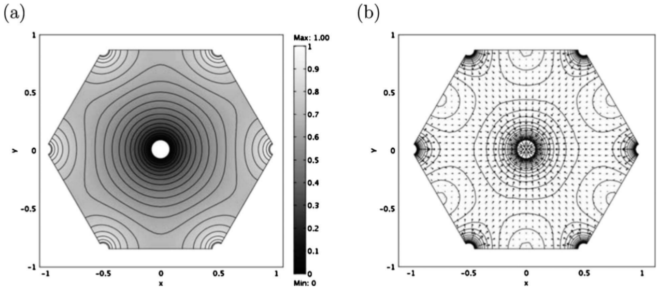


Fig. 2 (*a*) Pressure field, (*b*) velocity field (arrows), and contour lines of the velocity magnitude

Figure: Bonfiglio results

Consequences

- Scalable for HPC ($2 \cdot$ number of lobules)
- Produce problem with variable BC from problem with const BCs
- Cooperation with Lobule engineers from Taiwan (Regmed)
- Introduce **Lobule-field** for main characteristics
  For each point of domain correspond **Lobule**
- Lobule $\rightarrow \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{pmatrix}, \qquad \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \\ * \end{pmatrix}$
- Full geometry of Liver / Lack of Data

Deformable lobule model

$$\operatorname{div} \boldsymbol{\sigma} = 0$$

$$\boldsymbol{\sigma} = \frac{E\nu}{(1+\nu)(1-2\nu)}(\operatorname{grad} \boldsymbol{u})\boldsymbol{I} + \frac{E}{1+\nu}\boldsymbol{\varepsilon} - p_{\mathsf{eff}}\boldsymbol{I}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}((\operatorname{grad} \boldsymbol{u}) + (\operatorname{grad} \boldsymbol{u})^T)$$

Supplemented with following boundary conditions:

$$\boldsymbol{\sigma}_{nn} = -p_{\mathsf{eff}}, \quad \boldsymbol{x} \in \Gamma_p$$

$$\boldsymbol{u} = \boldsymbol{0}, \quad \boldsymbol{x} \in \Gamma_v$$

Displacement solution

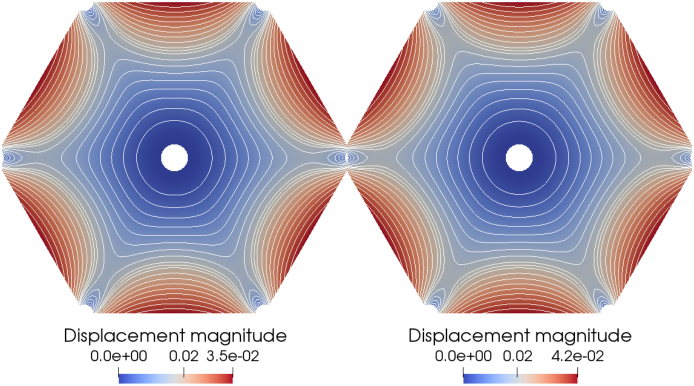


Figure: Deformation left for anisotropic case, right in isotropic case