The assessment of the thermal processes influence on liquid radioactive waste components transport at the “Severny” polygon

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II International conference «Multiscale methods and Large-scale Scientific Computing»

August 15-17, 2018 – INM RAS – Moscow, Russia

Plan

- Motivation
- Mathematical model
- Numerical procedure
- Verification
- Results of calculation for “Severny” injection polygon

Motivation

Radioactive decay can cause the self-heating of waste

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The heating influences fluid's behavior and radionuclides geomigration process

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Coupled ground-water flow, solute and heat transport model is needed

Model of coupled ground-water flow, solute and heat transport processes

$$\rho S \frac{\partial h}{\partial t} - \varphi \rho_0 \beta \frac{\partial T}{\partial t} + \varphi \sum_{i=1}^{N_{comp}} \kappa_{vol,i} \frac{\partial C_i}{\partial t} + \nabla(\rho \vec{u}) = \rho_s q_s,$$ – ground-water flow equation

$$\left[\varphi \rho_0 c^{\mathrm{f}} + (1-\varphi)\rho^{\mathrm{rock}} c^{\mathrm{s}}\right] \frac{\partial T}{\partial t} + \rho_0 c^{\mathrm{f}} \nabla\left(\vec{u} T\right) -$$
$$-\nabla\left[\left(\lambda + \varphi \rho_0 c^{\mathrm{f}} D_C\right) \nabla T\right] = q_s \rho_s c^{\mathrm{f}} T_s + W,$$ – heat transport equation

$$\varphi R_i \frac{\partial C_i}{\partial t} + \nabla\left(\vec{u} C_i\right) - \nabla\left(D \nabla C_i\right) = C_{s,i} q_s - \varphi R_i \Lambda C_i,\ i = 1, \ldots, N_{comp},$$ – solute transport equation

$$\vec{u} = -K\left(\nabla h + \frac{\rho - \rho_0}{\rho_0} \nabla z\right),$$ – Darcy's law

$$\rho = \rho_0 (1 - \beta(T - T_0)) + \sum_{i=1}^{N_{comp}} \kappa_{vol,i} C_i,$$ – fluid density

$$K = \frac{k \rho_0 g}{\mu(T, C)}$$ – tensor of hydraulic conductivity

Heat transport process. Volumetric heat source. Variable viscosity.

Heat-transport equation

$$\left[\varphi\rho_0 c^{\mathrm{f}} + (1-\varphi)\rho^{\mathrm{rock}} c^{\mathrm{s}}\right]\frac{\partial T}{\partial t} + \rho_0 c^{\mathrm{f}}\nabla\left(\vec{u}T\right) - $$
$$-\nabla\left[\left(\lambda + \varphi\rho_0 c^{\mathrm{f}} D_C\right)\nabla T\right] = q_s\rho_s c^{\mathrm{f}} T_s + W$$

- Thermal equilibrium
- Convection
- Conduction – thermal dispersion
- Wells
- Radiogenic heat

Variable viscosity

$$\mu(T) = A_1 \cdot A_2^{\left(\frac{A_3}{T+A_4}\right)}$$

Volumetric heat source

$$W = \sum_k Q^{(k)}\lambda^{(k)}\sum_{\beta}^{im,mo} C_\beta^{(k)}\left(\varphi_\beta + \rho_{bulk} k_{d_\beta}^{(k)}\left(C_\beta^{(k)}\right)\right)$$

$$Q^{(k)} = \frac{N_A}{M^{(k)}} E^{(k)}\left(1-\delta^{(k)}\right)$$

$\lambda^{(k)}$ - decay constant

$k_{d_\beta}^{(k)}$ - sorptivity coefficient

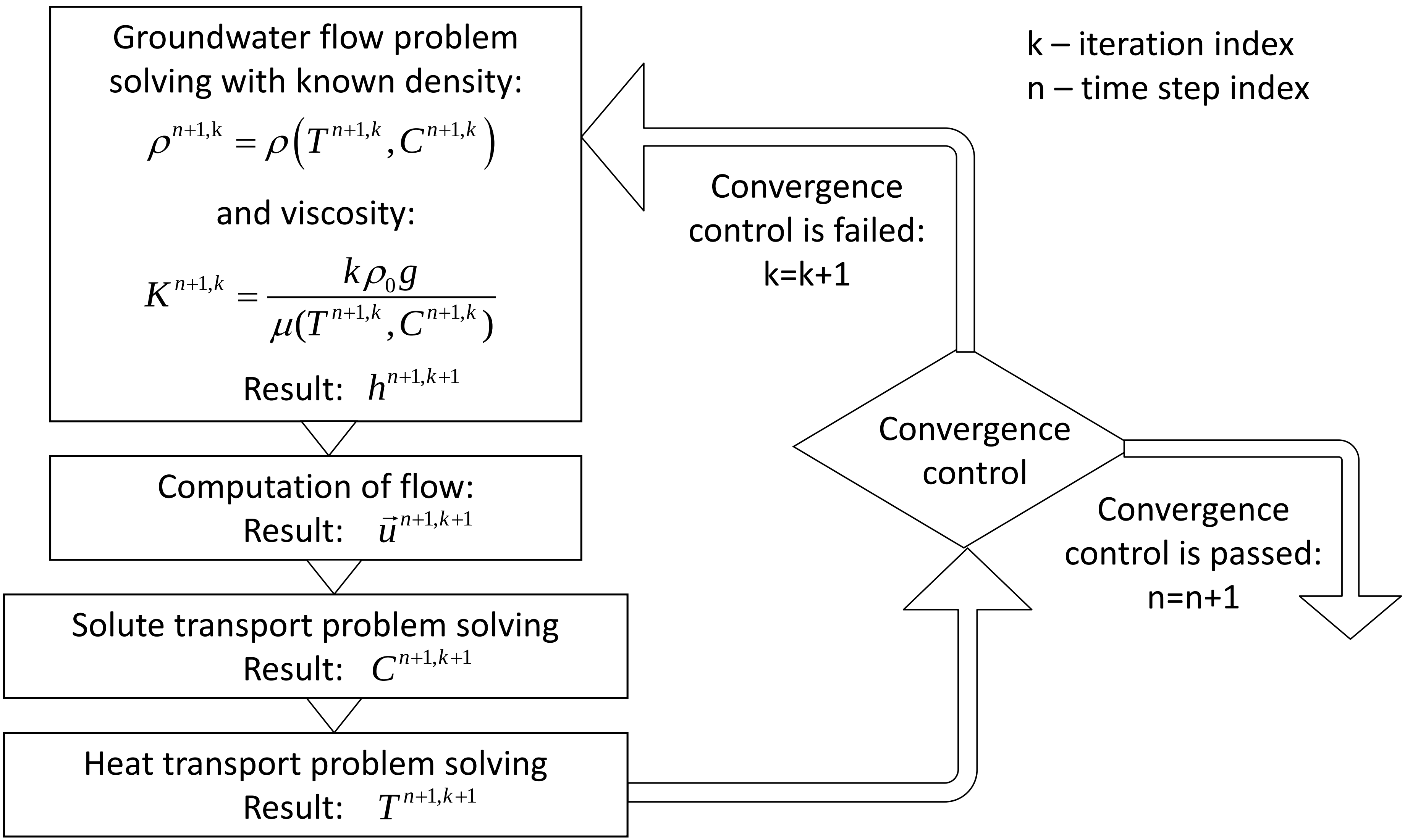
$E^{(k)}$ - heat emission per 1 decay event

$\delta^{(k)}$ - the proportion of neutrino energy

N_A - Avogadro’s number

$M^{(k)}$ - molar mass

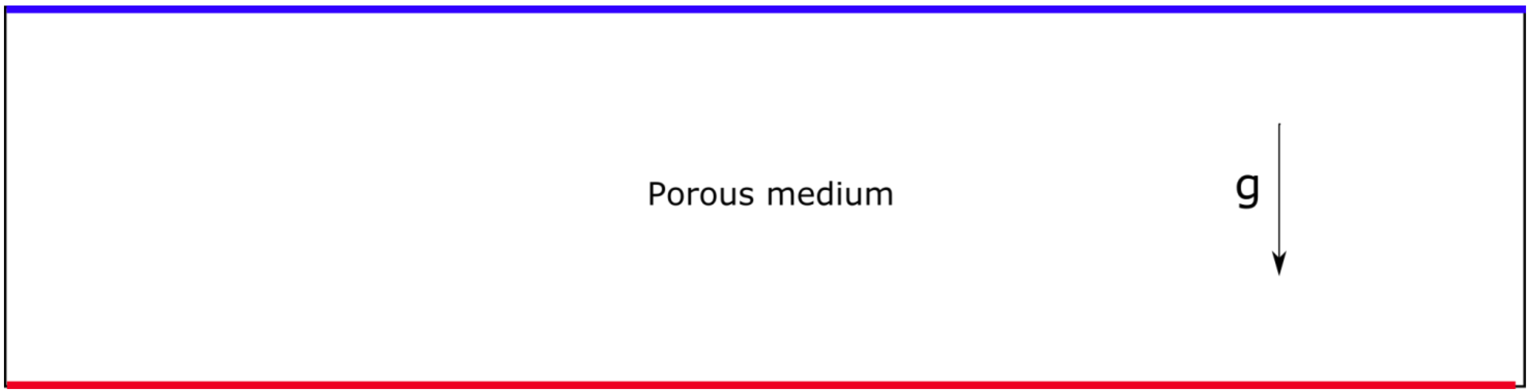
Numerical scheme: splitting method



Model verification

- Horton-Rogers-Lapwood (HRL) Convection
- Two-Dimensional Oil Convection in Aluminum Foam
- Natural Convection of Heat Generating Fluid

HRL Convection



Rayleigh-Darcy number

$$Ra = \frac{\beta \Delta T K H c^{\mathrm{f}} \rho}{\lambda}$$

Nusselt number

$$Nu = \frac{Q}{\lambda \frac{\Delta T}{H} LW}$$

H, L, W -sizes of area

β - temperature expansion coefficient

K - hydraulic conductivity

Q - Heat flow through the wall

c^{f} - fluid heat capacity

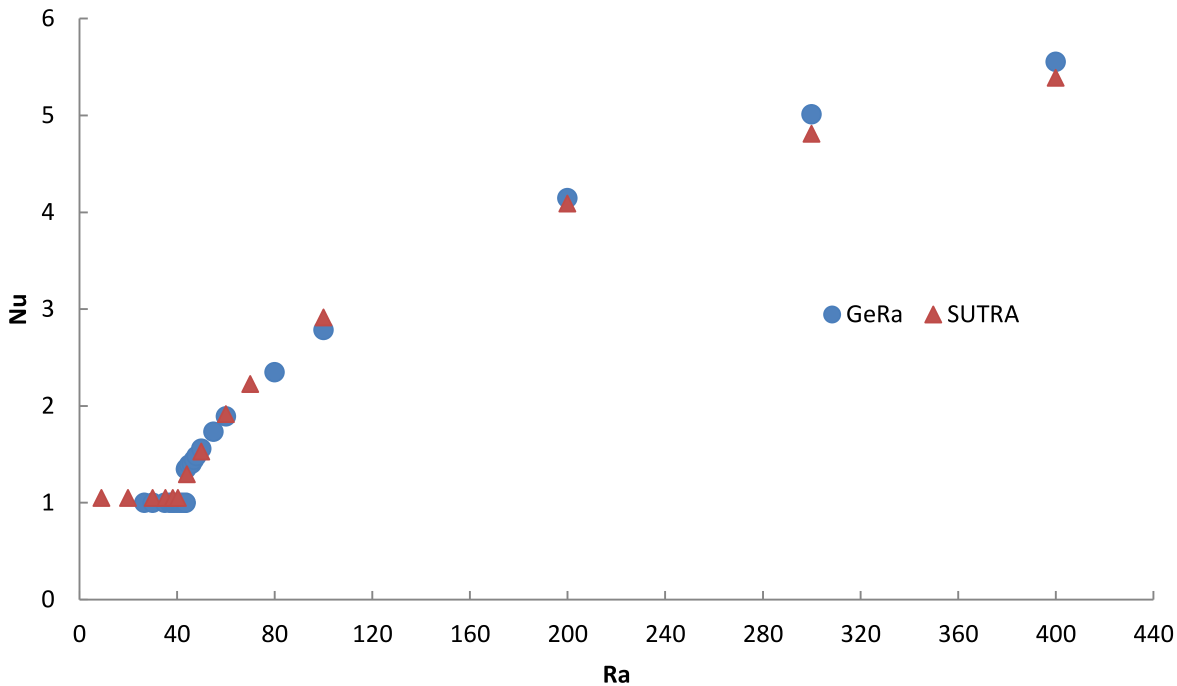
λ - heat conductivity

- Top: $T = 0$°C
- Bottom: $T = 1$°C
- Others: $\frac{\partial T}{\partial x} = 0$
- Flow BC: impermeability condition
- IC: $h = 0\ m$, linear temperature field

The problem was solved with different Rayleigh’s numbers. Nu – Ra relationship was built.

$$Ra > Ra_c = 4\pi^2$$ - transition to convective mode condition (according to analytical estimation)

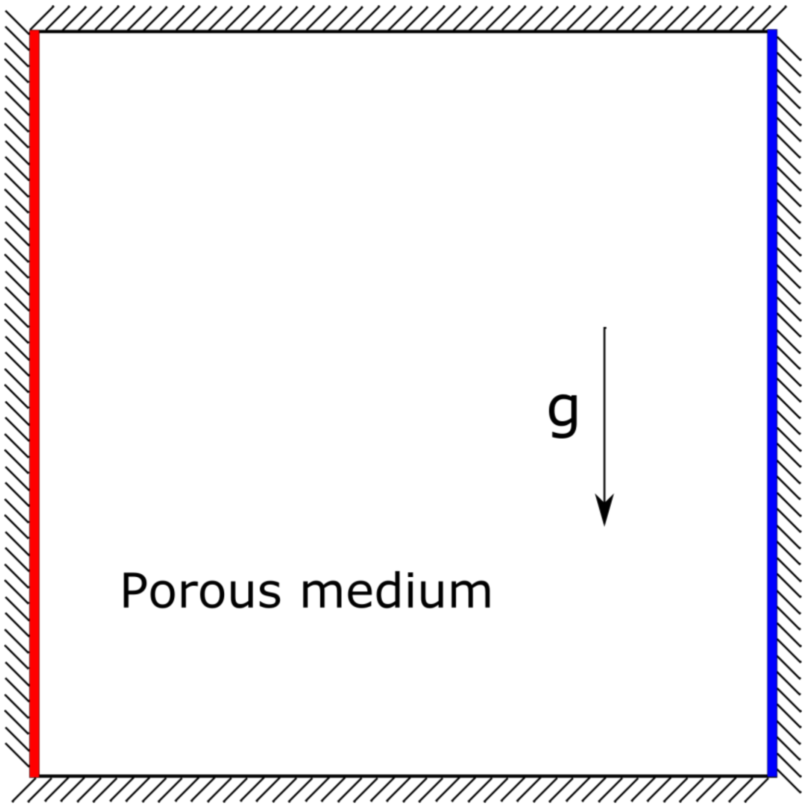
HRL Convection



- Nu – Ra relationship
- Crossverification with SUTRA code (*)
- Transition to convective mode: Ra=43.7

(*) Weatherhill, D., Simmons, C.T., Voss, C.E., and Robinson, N.I. Testing density-dependent ground-water models: twodimensional steady state unstable convection in infinite, finite and inclined porous layers // Advances in Water Resources. – 2004. – Vol. 27. – pp. 547-562.

Two-Dimensional Oil Convection in Aluminum Foam

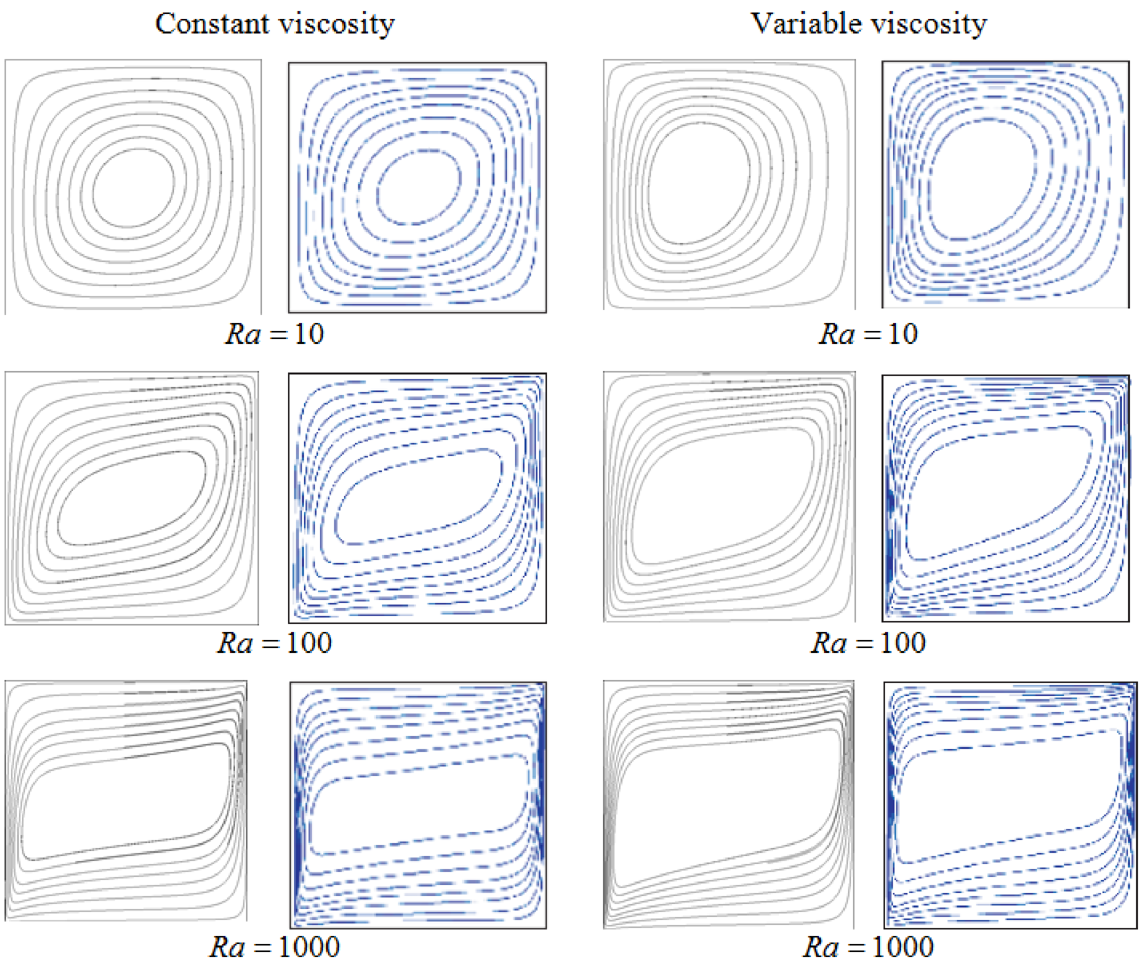


- Left: $T_{hot} = 36°C$
- Right: $T_{cold} = 6°C$
- Others: $\frac{\partial T}{\partial x} = 0$
- Flow BC: impermeability condition
- IC: $h = 0\ m$, linear temperature field

The problem was solved with different Rayleigh's numbers in two modes: constant viscosity case and variable viscosity case

For constant viscosity case the flow patterns are radially symmetric. For variable viscosity case the flow patterns are asymmetric: streamlines crowd together near the hot wall.

Two-Dimensional Oil Convection in Aluminum Foam



GeRa's and SEAWAT's v4 (*) results with different Rayleigh's numbers: streamlines

(*) Langevin, C.D., Thorne, D.T., Jr., Dausman, A.M., Sukop, M.C., and Guo, Weixing, 2007, SEAWAT Version 4: A Computer Program for Simulation of Multi-Species Solute and Heat Transport: U.S. Geological Survey Techniques and Methods Book 6, Chapter A22, 39 p

Natural Convection of Heat Generating Fluid

Buretta and Berman's convection cell(*):

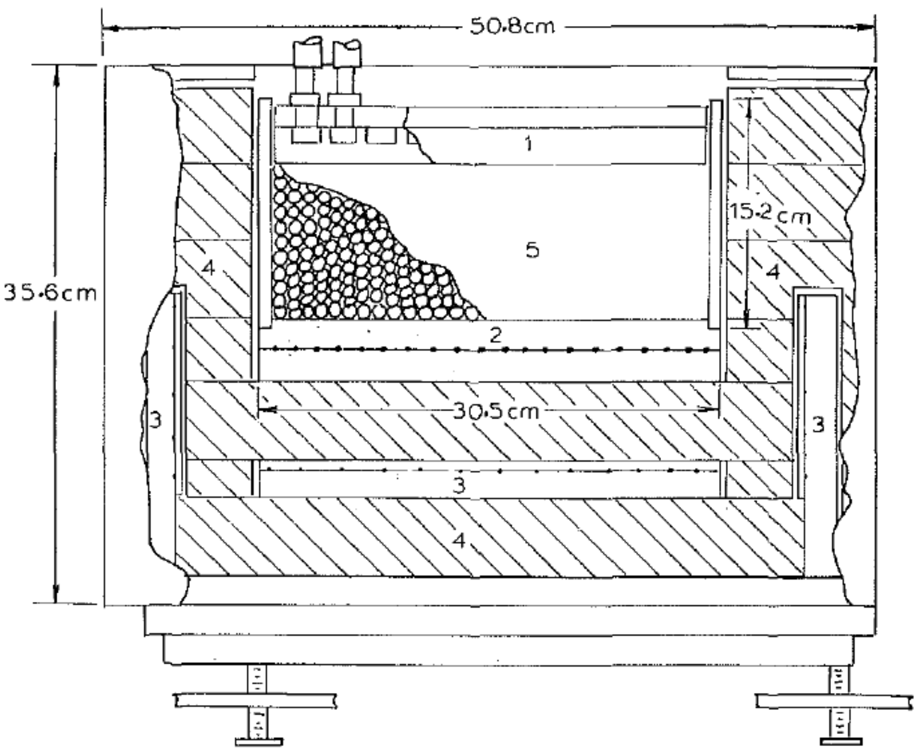
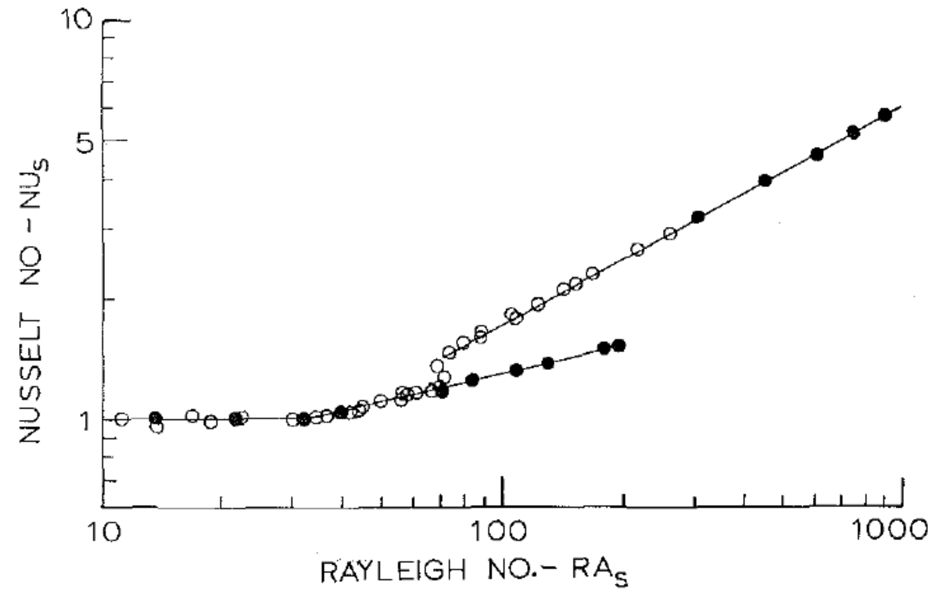


Fig. 1 Convection cell—1 Top plate; 2 Bottom plate; 3 Guard heaters; 4 Insulation; 5 Porous layer

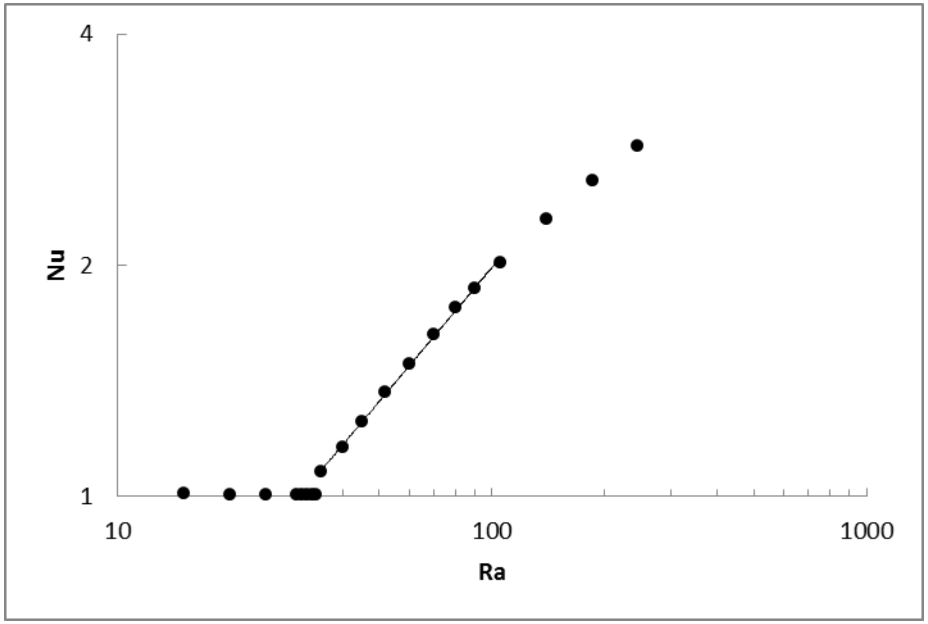


Nu – Ra experimental(*) relationship: two branches

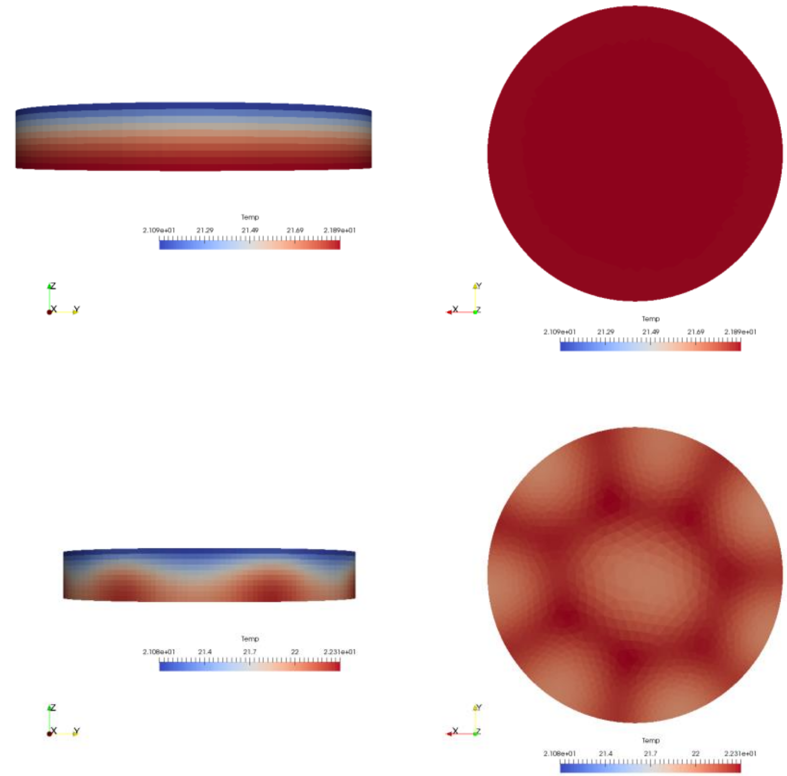
$$Ra_c^{exp} = 31.8$$

(*) Buretta, R. J., Berman, A. S. Convective heat transfer in a liquid saturated porous layer // ASME J. Appl. Mech. – 1976. – Vol. 43. – pp. 249–253.

Natural Convection of Heat Generating Fluid



Nu – Ra relationship from numerical experiment and approximation



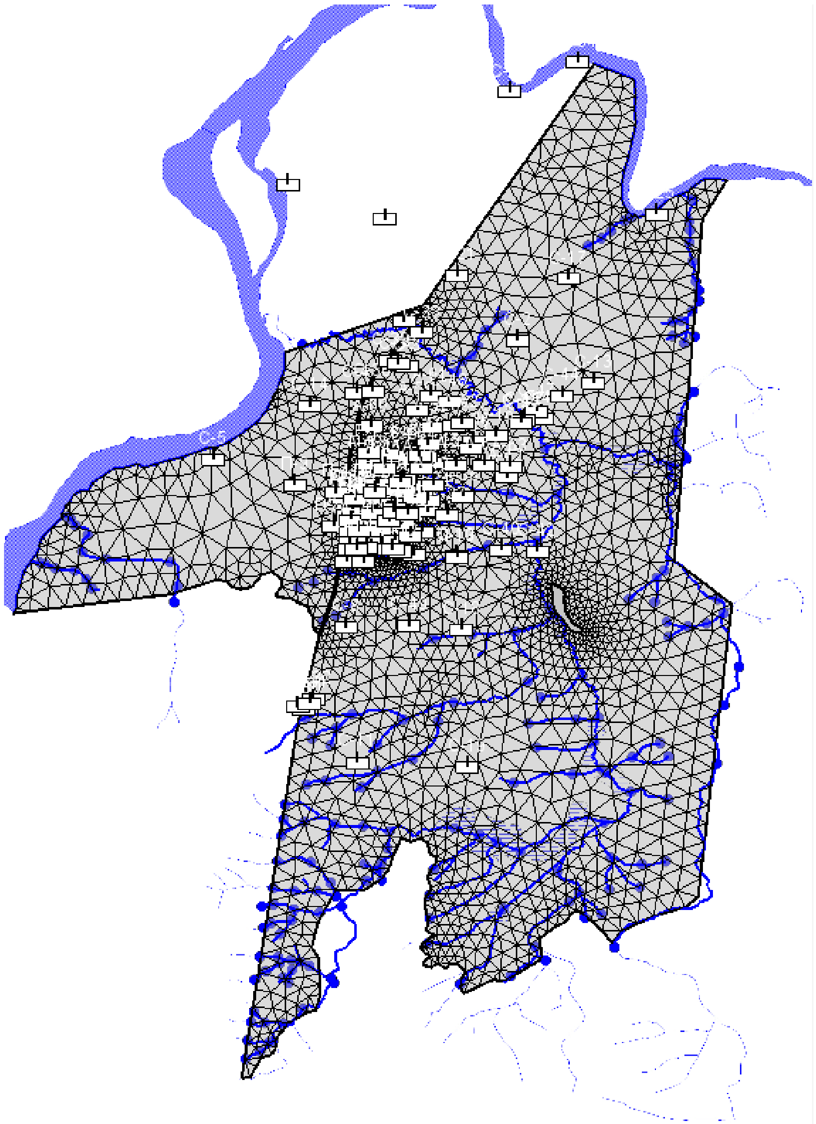
Temperature fields: numerical results for Ra = 25 (top) и Ra = 35 (bottom)

$$\text{GeRa:}\ \ \ln Nu = 0.5799 \ln Ra - 1.984$$

$$\text{Exp.:}\ \ \ln Nu = 0.553 \ln Ra - 0.871$$

$$Ra_c^{num} = 30.6$$

“Severny” polygon



- Lack of radionuclide composition data
- Short-term study: 50 years
- Maximum temperature: 200 °C
- Slight influence on geomigration

Conclusion

- Coupled ground-water flow, solute and heat transport model was implemented into the GeRa code
- It was verified on different tests
- “Severny” polygon is under investigations now

Thank you!