NUMERICAL SOLUTION OF NONSTATIONARY PROBLEMS FOR A SPACE-FRACTIONAL DIFFUSION EQUATION

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Many applied mathematical models involve both sub-diffusion (fractional in time) and super-diffusion (fractional in space) operators. Super-diffusion problems are treated as evolutionary problems with a fractional power of an elliptic operator. For example, suppose that in a bounded domain Ω on the set of functions $u(\boldsymbol{x}) = 0, \ \boldsymbol{x} \in \partial\Omega$, there is defined the operator $\mathcal{A}$: $\mathcal{A}u = -\triangle u, \ \boldsymbol{x} \in \Omega$. We seek the solution of the Cauchy problem for the equation with the fractional power elliptic operator:

$$\frac{du}{dt} + \mathcal{A}^\alpha u = f(t), \quad 0 < \alpha < 1, \quad 0 < t \le T,$$

$$u(0) = u_0,$$

for a given $f(\boldsymbol{x}, t)$, $u_0(\boldsymbol{x})$, $\boldsymbol{x} \in \Omega$ using the notation $f(t) = f(\cdot, t)$.
Standard two-level schemes are applied to solve numerically a Cauchy problem for an evolutionary equation of first order with a fractional power of the operator. The numerical implementation is based on the rational approximation of the operator at a new time-level. When implementing the explicit scheme, the fractional power of the operator is approximated on the basis of Gauss-Jacobi quadrature formulas for the corresponding integral representation. In this case, we have a Pade-type approximation of the power function with a fractional exponent. A similar approach is used when considering implicit schemes.