HOMOGENIZATION IN BIOLOGICAL PROBLEMS

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The talk is a review on the applications of the homogenization theory in multiscale mathematical modeling in biology with an accent at [2],[3], [12], [13], [11]. A large spectrum of biophysical models deals with viscous flows in porous media and in thin structures: blood flow in a network of vessels, blood flow through a fibrin binded RBC, network of capillaries (see [4] for rheology). In these models the standard homogenization techniques for the flows in porous medium can be applied. The most interesting problems concern the justified interface conditions between a Newtonian or non-Newtonian flow in some part of the domain and filtration in the porous part (see[10]). In particular in the Robin type junction conditions on the pressure was derived for the Stokes equation in a domain with periodic set of thin channels ([2]). On the other hand, modeling of the blood flow in a vessel needs to take into consideration the fluid-elastic (or viscoelastic) wall interaction, where the wall has a heterogeneous structure and can be homogenized (see [13]). Also the light absorption in a tissue is very different within blood vessels and out of vessels. Namely, it is much higher in vessels. This leads to a homogenization problem with contrasting coefficients, and the classical homogenization theory has limitations of applicability ([12]). Finally, complete asymptotic expansion of a solution was constructed in the case when classical homogenization doesn't work([3]). Modeling of wave propagation in the lungs via the homogenization is presented in [1], while for waves in the bones we refer to [5],[14]. An important direction is related to the multiscale modeling in electrophysiology with application to the heart motion. The main model there is the so called cable equation and it corresponds to a set of cells having conductive liquid part and weakly conductive but thin membrane. These problems were studied as formally [7] so that rigorously by means of Γ-convergence and two-scale convergence [15], [9],[6], [8]. Finally an important class of homogenization problems appears in application to the behaviour of cells, their nutrition, growth, death etc. Often the cells are modeled by discrete points, and so we deal with some differential equations with Dirac-like functions as the coefficients. One of such equations, diffusion discrete absorption (DDA) equation was homogenized in [11]. Currently this 1D equation is generalized for multiple dimensions. The work is supported by the Russian Science Foundation, grant number 14-11-00306, executed by National Research University Moscow Power Engineering Institute.

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