Homogenization of the Steklov spectral problem for the system of elasticity

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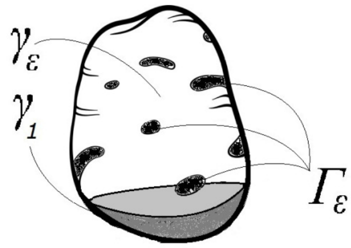
Let Ω be a smooth domain in $\mathbb{R}^d$, $d \geq 2$, and let $\partial\Omega$ be its boundary.



We suppose that $\partial\Omega = \gamma_1 \cup \gamma_\varepsilon \cup \Gamma_\varepsilon$ and consider the boundary value problem.

Consider the spectral problem

$$\begin{cases} L(u_\varepsilon^n) = 0 \ \text{ in } \ \Omega, \\ u_\varepsilon^n = 0 \ \text{ on } \ \gamma_1 \cup \gamma_\varepsilon, \\ \sigma(u_\varepsilon^n) = \lambda_\varepsilon^n u_\varepsilon^n \ \text{ on } \ \Gamma_\varepsilon, \ n = 1, 2, \dots \end{cases} \tag{1}$$

Here $u_\varepsilon^n \in (H^1(\Omega, \gamma_1 \cup \gamma_\varepsilon))^d$, $n = 1, 2, \dots$ The set $\{\lambda_\varepsilon^n\}$, $n = 1, 2, \dots,$ is the set of eigenvalues such that $\lambda_\varepsilon^1 \leq \lambda_\varepsilon^2 \leq \cdots \leq \lambda_\varepsilon^n \leq \cdots$, where the eigenvalues repeat according to their multiplicities. We suppose that

$$u_\varepsilon = (u_{\varepsilon,1}, \dots, u_{\varepsilon,d})^T,$$

$$L(u) = (L_1(u), \dots, L_d(u))^T := \frac{\partial}{\partial x_i}\big(A^{ij}(x)\frac{\partial u_\varepsilon}{\partial x_j}\big).$$

Here and throughout we assume the summation on the repeated indices.

A^{ij} are matrices $(d \times d)$ with elements a_{kl}^{ij}, which are bounded measurable functions, $a_{kl}^{ij}(x) = a_{lk}^{ji}(x) = a_{il}^{kj}(x)$,

$$\varkappa_1 \xi_{ki}\xi_{ki} \leq a_{kl}^{ij}(x)\xi_{ki}\xi_{lj} \leq \varkappa_2 \xi_{ki}\xi_{ki}, \ \ \varkappa_1, \varkappa_2 = const > 0, \ \ x \in \Omega, \tag{2}$$

where $\{\xi_{ki}\}$ are real symmetric matrices, $\nu = (\nu_1, \dots, \nu_d)$ is an outward normal vector to the boundary $\partial\Omega$, Γ_ε consists of the sets Γ_ε^m, $m = 1, \dots, N_\varepsilon$, the diameter of Γ_ε^m is less then or equals to ε, and the distance between them is greater than or equals to 2ε, where ε is a small positive parameter, $\gamma_\varepsilon = \partial\Omega \setminus \{\gamma_1 \cup \Gamma_\varepsilon\}$.

We study the limit behavior of eigenelements of problem (**??**), when ε tends to zero and $N_\varepsilon = O(|\ln\varepsilon|^{(1-\frac{\delta}{2})d-1}), 0 < \delta < 2 - \frac{2}{d}$. Thus N_ε is the number of Γ_ε^m on the boundary. The space $H^1(\Omega, \gamma_1 \cup \gamma_\varepsilon)$ is defined as the completion of the functions from the space $C^\infty(\overline{\Omega})$, vanishing in a neighborhood of $\gamma_1 \cup \gamma_\varepsilon$, with respect to the norm

$$\|v\|_{H^1(\Omega)} = \Big(\int\limits_\Omega \big(v^2 + |\nabla v|^2\big)dx\Big)^{\frac{1}{2}}.$$

Theorem 1. *There exists such a constant K independent of ε, that for eigenvalues λ_ε^n of the problem (**??**), the estimate*

$$\lambda_\varepsilon^n \geq K|\ln\varepsilon|^\delta$$

is valid for sufficiently small ε, where $0 < \delta < 2 - \frac{2}{d}, N_\varepsilon = O\big(|\ln\varepsilon|^{\left(1-\frac{\delta}{2}\right)d-1}\big)$ as $\varepsilon \to 0$.

References

[1] A. Chechkina, C. D'Apice, U. De Maio. Rate of Convergence of Eigenvalues to Singularly Perturbed Steklov-Type Problem for Elasticity System. *Applicable Analysis* DOI: 10.1080/00036811.2017.1416104