AUTOMATIC VARIATIONALLY STABLE DISCRETIZATIONS

VM Calo, N Collier, R Kirby, M Los,
A Niemi, M Paszynski, A Romkes, E Valseth
Curtin University

We discuss a class of discontinuous Petrov-Galerkin (DPG) methods for finite element (FE) approximations of boundary value problems for singularly-perturbed, second-order linear partial differential equations (PDEs). We will first introduce the DPG method with optimal basis functions as described by Demkowicz and Gopalakrishnan and adapted for advection-diffusion problems by Calo, Collier, and Niemi. We will then present a new DPG method which uses a first-order system weak representation of the governing PDEs. This new hybrid continuous-discontinuous DPG method uses classical continuous function spaces for the trial functions, to reduce computational cost and discontinuous function spaces for the test functions. The broken weighting spaces allow us to solve the optimal test functions locally at the element level by using the DPG philosophy. These weighting functions are optimal in the sense that they guarantee inherently stable FE approximations with best approximation properties in the energy norm. We compute the local test-function contributions numerically on each element with high accuracy without solving global variational statements. We will use 2D numerical verifications and convergence studies to validate our analysis. In particular, we will focus on convection-dominated diffusion problems with highly oscillatory (diffusion) coefficients.