MATHEMATICAL SIMULATION OF STEADY UNDERGROUND FILTRATION IN THE PRESENCE OF THE WELLS

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We consider the steady-state process of incompressible fluid underground filtration. This process follows a nonlinear multi-valued law of filtration with a limiting gradient in the presence of wells. Wells considered as point sources that described using Diracs delta-functions.

The problems of filtration of an incompressible fluid correspond to nonlinear filtration law has been considered previously. In this case generalized statements of these problems were formulated in the form of operator equations or variational inequalities in Hilbert spaces with pseudomonotone operators which acting in the case of linear growth of functions defining the filtration law from the Sobolev space $V = \overset{\circ}{W}{}_2^{(1)}$ conjugate with it and accordingly the situation is considered when a function describing the density of external sources or loads determines a linear continuous functional on V. Questions of the existence of solutions are investigated based on general theory of monotone operators.

In the work we consider problems with a less smooth right-hand side: in the non-dimensional case, the Dirac delta function modeling the load concentrated at a point does not belong to the space conjugate to V.

We note that for some linear problems as well as for a number of nonlinear problems in the case of classical domains, analytic solutions of elliptic boundary value problems with a delta function are well known. However, in the general case such solutions are unknown.

The above difficulty was eliminated through using additive allocation features related to the delta function.

Generalized statements of problems are formulated in the form of integral identities or integral variational inequalities with respect to a function $\overset{\circ}{W}{}_1^{(1)}$. An auxiliary problem was introduced with the right-hand side defined by the delta function. For the auxiliary problem solution is found in explicit form.

Generalized statements reduce to finding a solution of operator equations or variational inequalities in V. Just as in the smooth case, the properties of the operators entering into these operator equations or variational inequalities - boundedness, continuity, monotonicity, and coercivity are established. Which made possible to apply the known results of the theory of monotone operators to prove existence theorems. In this work proved that the sets of solutions of generalized problems are not empty, convex and closed. To solve the variational inequalities an iterative process is proposed which allows to simultaneously find both solutions of the problem and its gradient. This is very useful from a practical point of view.