NUMERICAL APPROXIMATION OF FRACTIONAL POWERS OF ELLIPTIC OPERATORS

B. Duan, R.D. Lazarov, and J.H. Pasciak
Texas A&M University

We shall discuss methods and algorithms for approximately solving the linear algebraic systems $L_h^\alpha u_h = v_h$, $0 < \alpha < 1$, for $u_h, v_h \in H_h$, H_h a finite dimensional Hilbert space. Such problems arise in finite element or finite difference approximations of problems $L^\alpha u = v$ with fractional powers of second order elliptic operators L. The algorithms are based on the method of Vabishchevich, that related the algebraic problem to a solution of a time-dependent parabolic type equation on the interval $[0, 1]$.

We develop and study two algorithms based on diagonal Padé approximation of the corresponding solution operator. The first one uses geometrically graded meshes in order to compensate for the singular behavior of the solution for t close to 0 for non-smooth data v. The second algorithm uses uniform in t meshes, but requires smoothness of the data v in order to retain optimal convergence rate. For both methods we estimate the error in terms of the number of time steps and the regularity of the data. Finally, we report some numerical experiments of finite element approximation of second order elliptic problems in one and two spatial dimensions.