Solution of the elliptic equation by a generalized multiscale finite element method in perforated and heterogeneous medium

Y. Efendiev[1], D.A. Spiridonov*[1] и M.V. Vasilyeva[1]

[1]North-Eastern Federal University, Yakutsk, Russia,
d.stalnov@mail.ru

In this work we consider the elliptic equation in perforated and heterogeneous medium. For perforated media, nonhomogeneous boundary conditions on perforations can be considered for many applied problems. For example, the Robyn boundary conditions usually applied at the boundaries of the solid medium for reactive processes in porous media. For such processes we should resolve perforations and heterogeneity using very fine grid which leads to the large discrete systems and computatioally expensive.

The classic method for solution of such problems on the coarse grid are a homogenization techniques [1]. The homogenization methods are used to construct the approximation of the problem on a coarse grid and allow to calculate the effective properties of the medium. In this method, an additional term is used for describing the pore-scale reaction on the coarse grid approximation. The multiscale methods can be applied for the coarse grid approximation by solution of the local problems and construction of the multiscale basis functions [2]. In this method we have a two-way information exchange between micro and macro levels. In this work, we use a generalized multiscale finite element method (GMSFEM) [3]. This method based on the calculation of the multiscale basis functions to reduce the dimension of the problem. For handling a non-homogeneous boundary conditions on the perforations or heterogeneous right hand side, in GMsFEM we construct the additional basis function, which improves the accuracy of the method. We present numerical comparision of the solution for the two-dimensional model problem in perforated and heterogeneous medium with non homogeneous Dirichlet, Neumann and Robin boundary conditions. The dependence of the accuracy of the method on the number of bases used is presented.

Список литературы

[1] Bakhvalov N. S., Panasenko G. P. Homogenization of Processes in Nonhomogeneous media. – 1984.

[2] Efendiev Y., Hou T. Y. Multiscale finite element methods: theory and applications. – Springer Science & Business Media, 2009. – T. 4.

[3] Eric T. Chung, Yalchin Efendiev, Guanglian Li, Maria Vasilyeva. Generalized multiscale finite element methods for problems in perforated heterogeneous domains // Applicable Analysis. Volume 95, Issue 10, 2016. Pp. 2254-2279.