MATHEMATICAL MODEL FOR THERMOELASTICITY PROBLEM IN FRACTURED POROUS MEDIA

{ GEORGII PROKOPEV & MARIA VASILYEVA} NORTH-EASTERN FEDERAL UNIVERSITY, YAKUTSK, RUSSIA

MULTISCALE MODEL REDUCTION AND HIGH PERFORMANCE COMPUTING LABORATORY

INTRODUCTION

We consider mathematical model describing the thermoelastic state of body, taking into account the presence of fractures. Mathematical model is described by the coupled system of equations for temperature and displacements. The standard way of accounting for fractures is a model based on the construction of a computational grid with a crack resolution through several cells and specifying heterogeneous coefficients.
Since in practice, the thickness of the crack is sufficiently small relative to the area size, its grid resolution leads to a significant increase in the unknowns. For the numerical solution of considered problem we make an approximation on coarse-scale grid using numerical homogenization procedure, which is based on computing effective properties for hererogeneous media. For the approximation of fine-scale grid for the fractured media with low conductive fractures, we use reduced model. We consider two types of the fractures: connected fracture network and unconnected small fractures.

MATHEMATICAL MODEL

Let us consider a porous material, the pores of which are filled with low conducting gas. The mathematical model, taking into account the heat exchange between the skeleton of the porous medium and the pore space, is written as follows

$$\begin{aligned} d_s \frac{\partial T_s}{\partial t} + \beta_s \operatorname{div} \frac{\partial u}{\partial t} - \operatorname{div}(b_s \operatorname{grad} T_s) + \zeta(T_s - T_g) = f, \quad x \in \Omega, \\ d_g \frac{\partial T_g}{\partial t} + \beta_g \operatorname{div} \frac{\partial u}{\partial t} - \operatorname{div}(b_g \operatorname{grad} T_g) + \zeta(T_g - T_s) = 0, \quad x \in \Omega, \\ - \operatorname{div}(\sigma(u)) + \beta_s \operatorname{grad} T_s + \beta_g \operatorname{grad} T_g = g, \quad x \in \Omega, \end{aligned} \tag{1}$$

where $\sigma(u) = \lambda \operatorname{tr}(\varepsilon)\mathcal{I} + 2\mu\varepsilon$, $d_s = (1-\phi)C_s$, $b_s = (1-\phi)k_s$, $d_g = \phi C_g$, $b_g = \phi k_g$, $f_s = (1-\phi)q_s$, $f_g = \phi q_g$ and ζ – function of heat transfer between the subregions. phi – porosity, k_s, C_s, q_s coefficients of thermal conductivity, heat capacity and sources for the skeleton of a porous medium, and k_g, C_g, q_g – coefficient of thermal conductivity, heat capacity and sources of pore space (gas-filled subdomain).
Here, we use an approach which is based on the use of double continuum models (double diffusion), which are constructed under the assumption that there is no local thermodynamic equilibrium, that is, $T_g \neq T_s$.
On the surface of a fracture, we set

$$\left\{-k_s \frac{\partial T_s}{\partial n_f}\right\} = \alpha_f [T_s], \quad \left[-k_s \frac{\partial T_s}{\partial n_f}\right] = 0, \quad x \in \gamma, \tag{2}$$

and

$$[\sigma n_f] = 0, \quad \{\sigma n_f\} = -\beta_s [T_s] n_f, \quad x \in \gamma. \tag{3}$$

We consider the approximation of the problem (1) using the Galerkin discontinuous method (IPDG, interior penalty discontinuous Galerkin).
The variational formulation of the problem is written as follows:

$$\begin{cases} m_T(T, r) + a_T(T, r) = l_T(r), \\ a_u(u, v) + g(T, v) = l_u(v), \end{cases}$$

where

$$\begin{aligned} a_T(T,r) = \int_\Omega k \nabla T \cdot \nabla r \, dx - \int_{\Gamma/\gamma} \{k \nabla T \cdot n\}[r] ds - \int_{\Gamma/\gamma} \{k \nabla r \cdot n\}[T] ds \\ + \frac{\theta}{h} \int_{\Gamma/\gamma} \{k\}[T][r] ds + \int_\gamma \alpha_f [T][r] ds + \int_{\partial\Omega} \eta \, T r \, ds, \end{aligned}$$

$$\begin{aligned} l_T(r) =& (f, r) + m_T(\check{T}, r) + \int_{\partial\Omega} \eta \, T_{out} r \, ds, \quad m_T(T, r) = \frac{1}{\tau}(T, r). \\ a_u(u, v) =& \int_\Gamma (\sigma(u), \varepsilon(v)) \, dx - \int_{\Gamma/\gamma} \{\tau(u)\}[v] ds - \int_{\Gamma/\gamma} \{\tau(v)\}[u] ds \\ &+ \frac{\theta}{h} \int_{\Gamma/\gamma} \{\lambda + 2\mu\}[u][v] ds + \int_\gamma \beta [T] n_f [v] ds, \\ g(T, v) =& \beta(\operatorname{grad}(T), v), \quad l_u(v) = (g, v), \end{aligned}$$

$\tau(u) = \sigma(u)n.$

This approximation allows us to take into account the interface conditions (2) – (3) in a natural way in the variational formulation of the problem.

HOMOGENIZATION

Next, we consider the numerical homogenization procedure for the heat transfer equation. Consider the parabolic equation

$$C(x)\frac{\partial T}{\partial t} - \nabla \cdot (k(x) \nabla T) = 0, \quad x \in \Omega, \quad t > 0,$$

where $a(x)$ and $b(x)$ – heterogeneous coefficients defined on a fine-scale grid.
To construct an effective diffusion properties, we will solve local problems in cells of the coarse grid K:

$$-\nabla \cdot (k(x) \nabla m_l) = 0, \quad x \in K,$$

with the Dirichlet boundary conditions

$$m_l = x_l \quad x \in \partial K.$$

The effective diffusion coefficient $k^\star$ is calculated by averaging the fluxes

$$k^\star_{\cdot,l} = \frac{1}{|K|} \int_K k(x) \nabla m_l \, dx.$$

To average the coefficient $b^\star$ with the time derivative, since it does not stand under the sign of the derivative, you can use the average value in the local area

$$b^\star = \frac{1}{|K|} \int_K b(x) \, dx.$$

After calculating the averaged coefficients $k^\star(x)$ and $b^\star(x)$, we can solve the problem on a coarse-scale grid

$$b^\star(x)\frac{\partial T}{\partial t} - \nabla \cdot \left(k^\star(x) \nabla T\right) = 0, \quad x \in \Omega, \quad t > 0,$$

where $k^\star$ is an anisotropic diffusion coefficient

$$k^\star = \begin{bmatrix} k^\star_{x,x} & k^\star_{x,y} \\ k^\star_{y,x} & k^\star_{y,y} \end{bmatrix}.$$

To determine the effective modulus of elasticity for each coarse grid cell K, we solve the equation on a shallow grid in the local area, for example, using the finite element method. We write the variational formulation: find $u^{(rs)} \in V_h(K)$ such that

$$a(u^{(rs)}, v) = 0, \quad v \in V_h(K),$$

where

$$a(u, v) = \int_K (\sigma(u), \varepsilon(v)) dx,$$

with the Dirichlet boundary conditions

$$u^{(rs)} = \Lambda^{(rs)} x \quad \text{on } \partial K,$$

$$\Lambda^{(rs)}_{ij} = \frac{1}{2}\left(\delta_{ir}\delta_{js} + \delta_{is}\delta_{jr}\right)$$

and $r, s = 1, 2$. The components of the effective modulus of elasticity $C^\star$ are defined as the mean value of locally calculated strains

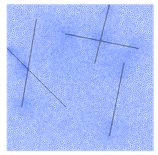
$$C^\star_{rspq} = \frac{1}{|K|} \int_K C_{ijkl} \varepsilon^{(rs)}_{ij} \varepsilon^{(pq)}_{kl} \, dx,$$

where $r, s, p, q = 1, 2$ and $\varepsilon^{(rs)} = \varepsilon(u^{(rs)})$ – strain tensor for (rs) load.

Distributing displacement on coarse element with fracture.

RESULTS

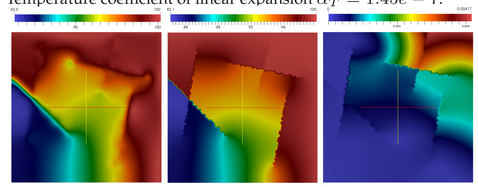
Let us consider numerical simulation of the thermoelasticity problem using the proposed models. The geometric area with the calculated grid is shown in Figure below. The computational domain $\Omega = [0, L_x] \times [0, L_y]$ c $L_x = L_y = 1$. The simulation was performed with the initial condition $T_0 = 10$ with $\tau = 360$ and $t_{max} = 36000$.



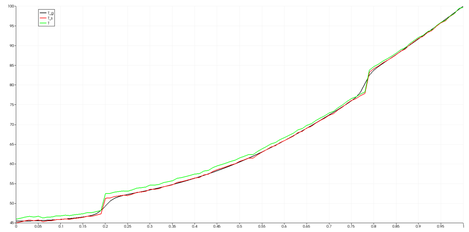
Computational grid with 19,800 elements.

For the calculations, the following parameters were used:

- Heat capacity coefficients for the main medium and pores: $c_s = 6e^5$ and $c_g = 10^3$,
- Coefficients of thermal conductivity of the main medium and pores: $k_s = 15$ and $k_g = 0.02$,
- fracture properties: $k_f = 0.02$ (coefficient of thermal conductivity), $\delta = 0.001$ (fracture thickness) and $\alpha_f = k_f/\delta$,
- source $f = 10^8$,
- Mass transfer coefficient between continuum $\zeta = 50$.
- Lame parameters: $\lambda = 1.25e11$ and $\mu = 0.8e11$.
- Temperature coefficient of linear expansion $\alpha_T = 1.45e-7$.



Distribution of temperature fields for two continuum and displacement at the last moment of time using the double diffusion model for the first test geometry.



Temperature distribution at the last time along the line $y = L_y/2$ for a model using one and two continuum for the first test geometry.

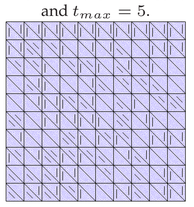
Next, let us consider numerical simulation of the thermoelasticity problem using the proposed models. The geometric area with the calculated grid is shown in Fig.

For numerical comparision, we consider the relative L^2 error

$$||e||_{L^2} = \frac{\sqrt{\int_\Omega (T_h - T_H)^2 dx}}{\sqrt{\int_\Omega T_h^2 dx}}$$

where T_h and T_H are the fine-scale and coarse-scale solutions, respectively.

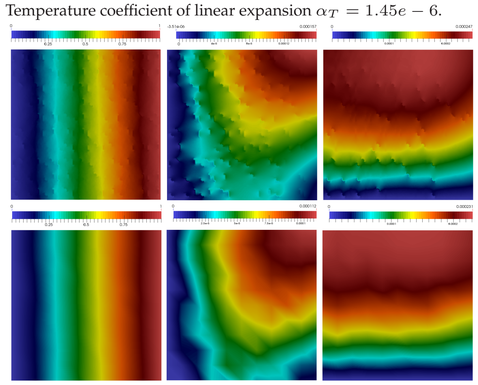
. The simulation was performed with the initial condition $T_0 = 0$ with $\tau = 0.1$ and $t_{max} = 5$.



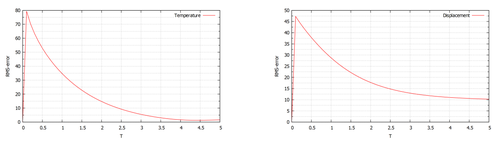
The fine-scale grid with 100,000 elements.

For the calculations, the following parameters were used:

- Heat capacity coefficients for the medium: $c = 1$,
- Heat conductivity coefficients of the medium $k = 0.1 + 2000 \cdot \sigma(x)$,
- Crack properties: $k_f = 0.01$ (coefficient of thermal conductivity), $\delta = 0.001$ (fracture thickness) and $\alpha_f = k_f/\delta$,
- source $f = 0$,
- Lame parameters: $\lambda = 1.25e5$ and $\mu = 0.8e5$.
- Temperature coefficient of linear expansion $\alpha_T = 1.45e - 6$.



Distribution of temperature and displacement fields at $T = 5.0$ time for fine and coarse grids.



Dependents of RMS error from time. Left: for temperature. Right: for displacement.

REFERENCES

[1] Katja K Hanowski and Oliver Sander. Simulation of deformation and flow in fractured, poroelastic materials. 2016

[2] J Kim, HA Tchelepi, R Juanes. Stability and convergence of sequential methods for coupled flow and geomechanics: Drained and undrained splits. 200(23):2094–2116, 2011.

[3] NS Bakhvalov and GP Panasenko. Homogenization in periodic media, mathematical problems of the mechanics of composite materials. ed: Nauka, Moscow, 1984.

CONCLUSION

In this work, we study an Interior penalty discontinuous Galerkin method for a fine-scale solution of the thermoelasticity problem.
Computing of effective properties is based on solution of the local problems in each coarse mesh cells. We present numerical result, where we illustrate the accuracy of the proposed algorithm. Our results show that the presented method can provide good accuracy of the solution with small fractures.

CONTACT INFORMATION

Email khloros35@gmail.com