Multiscale model reduction for coupled heterogeneous problems

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Mathematical modeling and numerical solution of the applied problems in highly heterogeneous media is a challenging task. A classical numerical method for solving such problems should use a computational grid that resolves all small heterogeneities. For such problems, a numerical homogenization or multiscale methods are used. For the processes in a periodic heterogeneous media, where the length of the period is small compared to the domain size, the asymptotic homogenization methods is used to construct a coarse-scale approximation and calculate effective properties that take into account small scale heterogeneities. Nowadays, a multiscale methods are becoming popular, for example, heterogeneous multiscale method (HMM), multiscale finite element method (MsFEM), variational multiscale method (VMS) and others. Multiscale methods should combine the simplicity and efficiency of a coarse-scale model, and the accuracy of microscale approximations.

In this work, we construct a reduced order model using the Generalized Multiscale Finite Element Method (GMsFEM). This method involves two basic steps: (1) the construction of multiscale basic functions that take into account small scale heterogeneities in the local domains and (2) the construction of the coarse scale approximation [1]. For the construction of the multiscale basic functions, we solve spectral problems in local domains. Spectral problems help identifying the most important characteristics of the solution. In contrast to the available techniques, this method allows avoiding the limitations associated with idealization and limitations on the applicability of the method [2]. This method also is more general technology that takes into account the different scale processes. The construction of basic functions occurs independently for each local domain, doesn't require the exchange of information between processors, and has a high parallelization efficiency. Using constructed multiscale basic functions, we construct a mathematical model on a coarse grid that allows significantly reducing the computation time, the amount of used memory, and can be used to perform calculations for a given configuration of heterogeneous properties.

We consider several applied problems with different types of heterogeneities. As a first problem, we consider gas filtration in the fractured poroelastic medium and present multiscale method for solution of the problem on a coarse grid [3]. Next, we consider wave propagation in the elastic fractured media [4], and finally present multiscale method for the solution of the pore-scale electrochemical processes in the litium-ion batteries.

References

[1] Y. Efendiev, J. Galvis, and T. Hou, Generalized multiscale finite element methods, Journal of Computational Physics, 251, pp. 116-135, 2013.

[2] E. T. Chung, Y. Efendiev, G. Li, and M. Vasilyeva, Generalized multiscale finite element method for problems in perforated heterogeneous domains, Applicable Analysis, 255 (2015), pp. 1-15.

[3] I Y Akkutlu, Y. Efendiev, and M. Vasilyeva. Multiscale model reduction for shale gas transport in fractured media. Computational Geosciences , 20(5), pp. 953973, 2016.

[4] E. T. Chung, Y.Efendiev, R. L.Gibson, and M. Vasilyeva. A generalized multiscale finite element method for elastic wave propagation in fractured media. GEM-International Journal on Geomathematics, 7(2), pp. 163-182, 2016.