ON ONE MULTISCALE PROBLEM: MODEL OF WELLS FOR RESERVOIR SIMULATION

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INTRODUCTION

In this poster, a study of a problem with wells for which nonlocal boundary conditions are given. It is shown that the problem is equivalent to a mixed problem without wells. For this formulation, an error estimate of a mixed finite element method in the 2D case is studied. Also, a numerical study of a diffusion problem in the presence of wells on which integral boundary conditions are used is performed. It is shown that a method proposed earlier is fully efficient and offers certain advantages as compared with direct modeling of wells based on the finite element method. The results of calculations for two wells are presented.

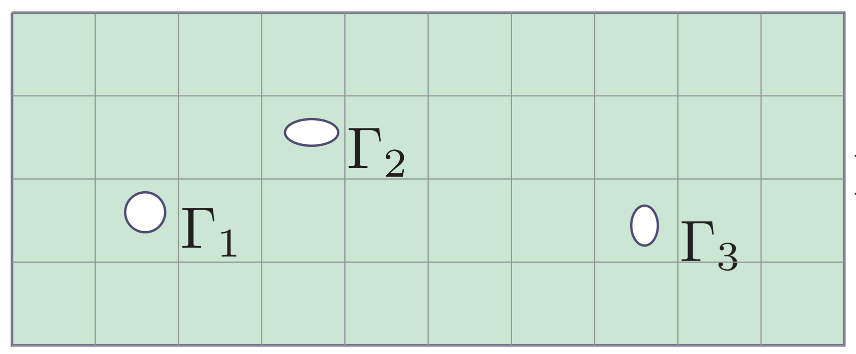
MATHEMATICAL MODEL

Single-phase incompressible fluid filtration problem

$\nabla \cdot a(\boldsymbol{x})\nabla p = 0, \qquad \boldsymbol{x} \in \Omega_0$ (green)

$p = 0, \qquad \boldsymbol{x} \in \Gamma_0$

$p = c_l, \quad \int\limits_{\Gamma_l} a\nabla p \cdot \boldsymbol{n}\, d\gamma = Q_l, \quad l = 1, ..., L$



Find $p \in L_{2,c}(\Omega) \quad \boldsymbol{u} \in \boldsymbol{H}_{div,c}(\Omega)$

$\forall v \in H_{div,c}(\Omega) \quad \int\limits_{\Omega_0} \frac{1}{a}\boldsymbol{u} \cdot \boldsymbol{v} dx - \int\limits_{\Omega} p\nabla \cdot \boldsymbol{v} dx = 0$

$\forall q \in L_{2,c} \quad \int\limits_{\Omega} q\nabla \cdot \boldsymbol{u} dx = \sum\limits_{l=1}^{L} Q_l q_l$

Properties:

- Solution exists

  $\|\boldsymbol{u}\|^2_{L_2(\Omega_0)} + \|\nabla \cdot \boldsymbol{u}\|^2_{L_2(\Omega)} + \|p\|^2_{L_2(\Omega)} \le C \sum\limits_{l=1}^{L} \frac{1}{mes(\Omega_l)} Q_l^2$

- Velocity is determined up to solenoidal functions
- $p \in L_{2,c}(\Omega) \cap H_0^1(\Omega) \Rightarrow$

  $\|\boldsymbol{u}\|^2_{L_2(\Omega_0)} + \|p\|^2_{L_2(\Omega)} \le C \sum\limits_{l=1}^{L} \frac{1}{mes(\Omega_l)} Q_l^2$

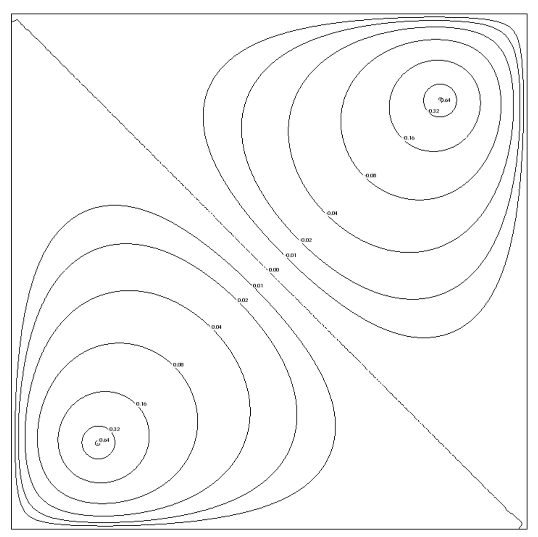
NUMERICAL RESULTS

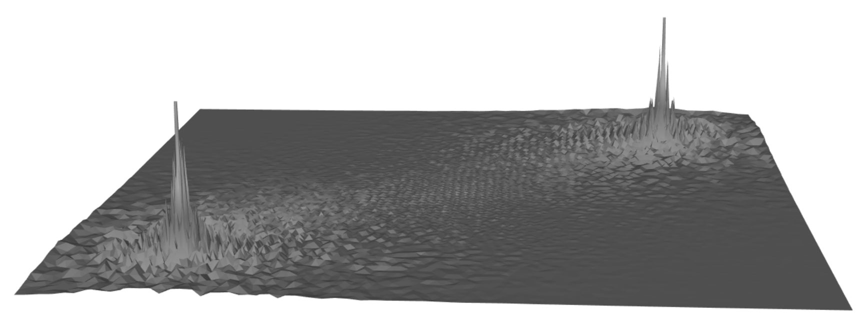
Features of the method

- Main concept is continue solution inside wells (Fictitious domain method)
- So we can replace well production by cell production
- Finite element realisation based on using minimal degree Raviart-Thomas elements

Error compared with direct computation based on unstructured mesh refinement near wells

$r_w \backslash h$	$3.70 \cdot 10^{-2}$	$1.23 \cdot 10^{-2}$	$4.12 \cdot 10^{-3}$
$6.17 \cdot 10^{-3}$	0.0159	N / A	N / A
$2.06 \cdot 10^{-3}$	0.0238	0.0050	N / A
$0.69 \cdot 10^{-3}$	0.0291	0.0067	0.0017





Left — isolines of solution, Right — abs(error) field

CONCLUSION

- A new method was proposed
- The calculations have shown higher order of accuracy in the grid spacing than that established earlier theoretically
- The method does not require refinement of the calculation grid for the wells and, hence, it can be used with a regular rectangular grid
- The method can be generalized to the three-dimensional case and this approach can be used for problems of multiphase filtration

OTHER PROBLEM

Extension on two-phase fluid flow

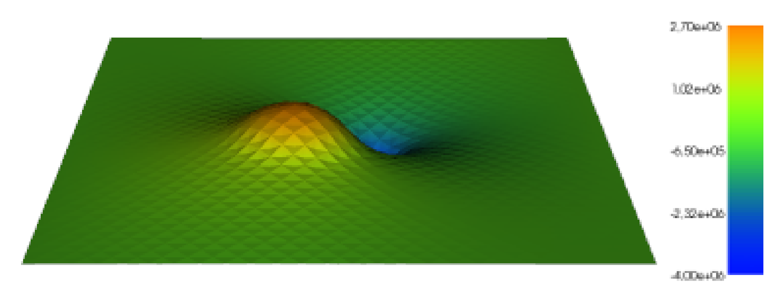
$p(t), s(t) \in L_{2,c}(\Omega), \quad \boldsymbol{u}(t), \boldsymbol{u}_w(t) \in \boldsymbol{H}_{div,c}(\Omega)$

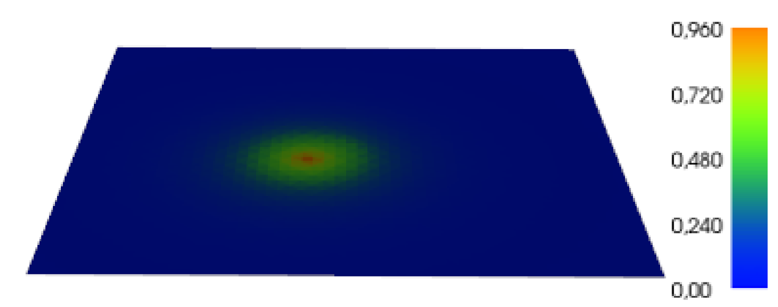
$\forall \boldsymbol{v} \in \boldsymbol{H}_{div,c}(\Omega) \qquad \int\limits_{\Omega_0} \frac{1}{k(s)}\boldsymbol{u} \cdot \boldsymbol{v} dx - \int\limits_{\Omega} p\nabla \cdot \boldsymbol{v} dx = 0$

$\forall q \in L_{2,c}(\Omega) \qquad \int\limits_{\Omega} q\nabla \cdot \boldsymbol{u} dx = \sum\limits_{l=1}^{L} Q_l q_l$

$\forall \boldsymbol{v} \in \boldsymbol{H}_{div,c}(\Omega) \qquad \int\limits_{\Omega_0} \boldsymbol{u}_w \cdot \boldsymbol{v} dx - \int\limits_{\Omega_0} \frac{k_w(s)}{k(s)} \boldsymbol{u} \cdot \boldsymbol{v} dx = 0$

$\forall q \in L_{2,c}(\Omega) \qquad \int\limits_{\Omega_0} m\frac{\partial s}{\partial t} q dx + \int\limits_{\Omega} q\nabla \cdot \boldsymbol{u}_w dx = \sum\limits_{l=1} L_w Q_l q_l$





Left — pressure, Right — saturation

REFERENCES

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