Numerical Simulation of Deformations of Softwood Sawn Timber

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Introduction

The development of timber cutting technology is tightly linked with sawn material quality and therefore it cost. Timber cutting pattern determines inner stress distribution and hence, it deformations, which is one of the aspects of quality of the timber [1, 2]. In Fig. 1 some examples of cutting pattern are presented.

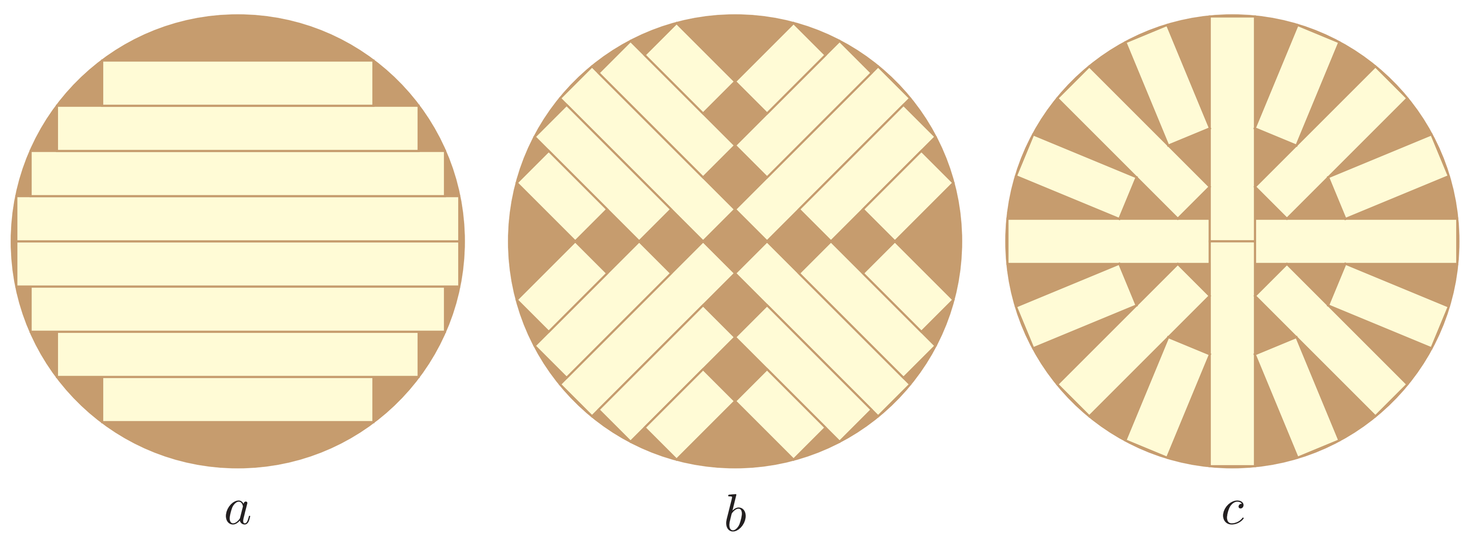

Figure 1: Plain(a), Quarter(b) and Rift(c) cutting patterns

In order to present the most robust way of determination of timber cutting pattern one must produce accurate mathematical model, which can predict a real timber stress-strain state. The main goal of investigation is to evaluate maximum value of bow and crook deformation as these values directly affect timber quality and to verify empirically derived expression for inner stress components by numerical simulation. These types of timber warping are explained in Fig. 2.

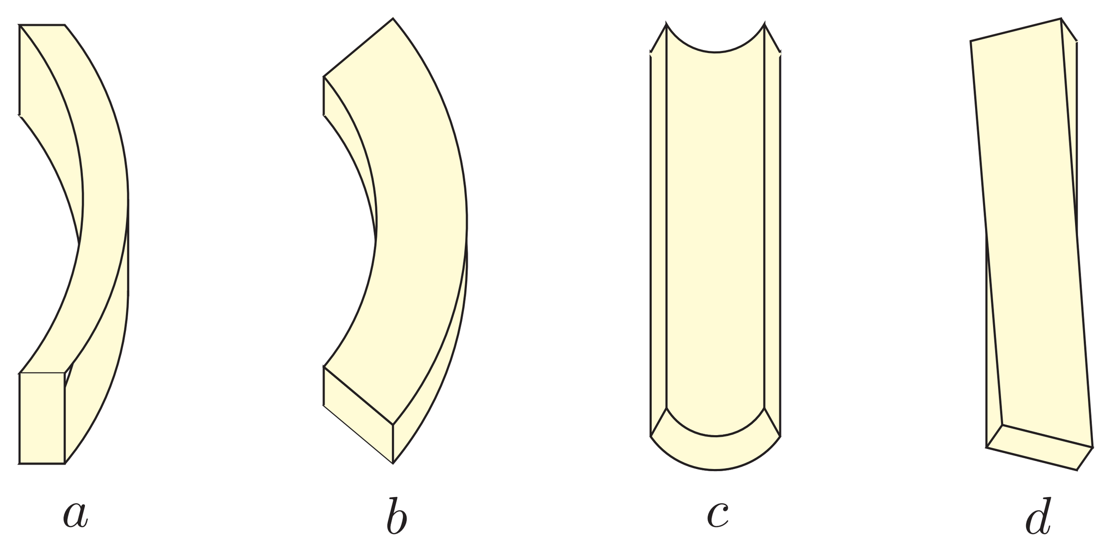

Figure 2: Types of timber warping: bow(a), crook(b), cup(c), twist(d)

In this work we consider numerical simulation of mathematical model for dahurian larch wood, which accounts for great part of timber export of Yakutia. Dahurian larch refers to coniferous wood species with a noticeable core zone. Heartwood and sapwood have different physical and mechanical characteristics. That affects on the formation and distribution of internal stresses during growth and subsequent processing of wood.

[1] Akopyan A. L. Glukhikh, V.N. *Nachal'nye naprjazhenija v drevesine*. The Ministry of Education and Science of the Russian Federation, 2016.

[2] Khrabrova O.YU. Glukhikh, V.N. Bending of sawn wood products obtained from conventional sawing and parallel to generatix sawing. *Architecture and engineering*, 1(1):4–9, 2016.

Mathematical model

Let us consider mathematical model that describes stress-strain state in computational domain Ω, which refers to sawn timber without body force

$$\operatorname{div} \boldsymbol{\sigma}(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \Omega. \tag{1}$$

Then we add relation between stress tensor $\boldsymbol{\sigma}$ and deformation tensor ε for orthotropic material defined in local rotated coordinate system (r, t, a):

$$\boldsymbol{\sigma}(\boldsymbol{x}) = \boldsymbol{C}\varepsilon(\boldsymbol{x})$$

in order to define equation for displacement vector. Here

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix}$$

is symmetric matrix with following components

$$C_{11} = \frac{1-\mu_{at}\mu_{ta}}{\Delta E_t E_a}, \quad C_{22} = \frac{1-\mu_{ar}\mu_{ra}}{\Delta E_r E_a}, \quad C_{33} = \frac{1-\mu_{rt}\mu_{tr}}{\Delta E_r E_t},$$
$$C_{12} = \frac{\mu_{tr}+\mu_{ar}\mu_{ta}}{\Delta E_t E_a}, \quad C_{13} = \frac{\mu_{ar}+\mu_{tr}\mu_{at}}{\Delta E_t E_a}, \quad C_{23} = \frac{\mu_{at}+\mu_{rt}\mu_{ar}}{\Delta E_r E_a},$$
$$C_{44} = G_{rt}, \quad C_{55} = G_{ra}, \quad C_{66} = G_{ta},$$

where

$$\Delta = \frac{1-\mu_{rt}\mu_{tr}-\mu_{ra}\mu_{ar}-\mu_{ta}\mu_{at}-2\mu_{tr}\mu_{at}\mu_{ra}}{E_r E_t E_a}.$$

where E_r, E_t, E_a are radial, tangential and longitudinal Young's moduli, G_{ij}, μ_{ij} are Shear moduli and Poisson's ratio, respectively.
The object of investigation is dahurian larch sawn timber with 156mm × 54mm × 5m in size. The wood, from which timber has been sawn, is modeled as conoid. According to previous investigations[1] we present following inner stress distribution model for dahurian larch

$$\sigma_r(r) = \sigma_{ta} \ln\left(\frac{r}{R}\right),$$
$$\sigma_t(r) = \sigma_{ta}\left(1 + \ln\left(\frac{r}{R}\right)\right),$$
$$\sigma_a(r) = \sigma_0\left(1 - 7\frac{r}{R} - 8\frac{r^{14}}{R^{14}}\right).$$

Results

As an example of numerical simulation of model problem we consider sawn timber with long side parallel to longitudinal axis of the tree. Location of timber inside wood shown as cut in Fig. 3. For simulation we used typical values for $\sigma_0 = 2MPa$ and $\sigma_{ta} = 0.1MPa$.

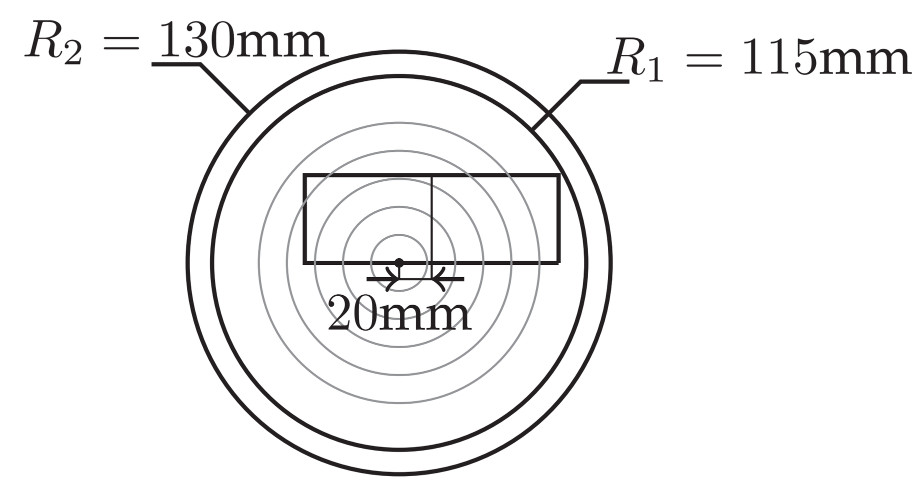

Figure 3: Timber location inside the wood

For these values we receive displacement distribution shown in Fig. 4

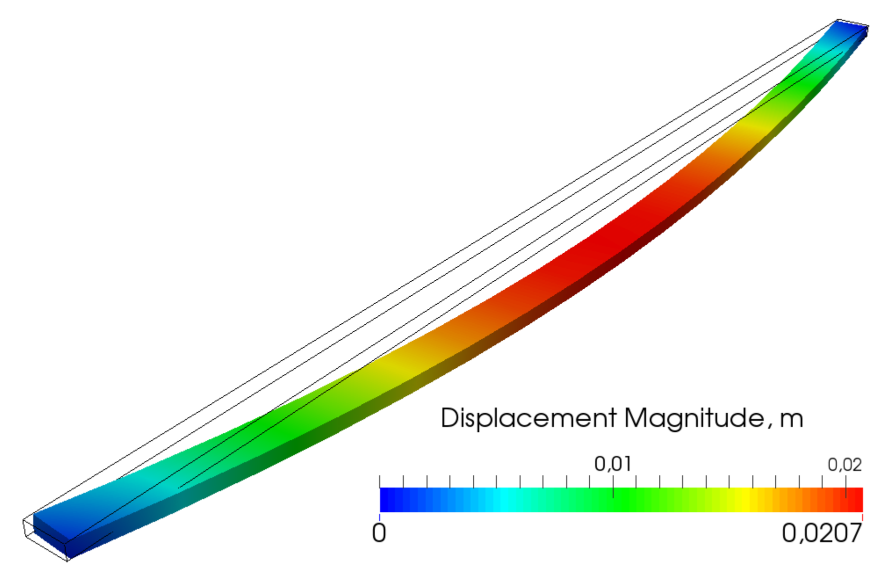

Figure 4: Displacement magnitude distribution. Shape warped by 10× displacement vector

As it is shown in Fig. 4, for the placement of sawn timber we have strong bow and crook deformations. These values according to GOST26002-83 standard refers to 5th quality grade, which is unlikely to be exported. Cup and twist deformations are nearly undetectable.

Conclusion

We accomplish numerical solution for linear elasticity problem for sawn timber, which has anisotropic elastic nature. As a result, for typical values we received adequate distribution of displacement.

Future Research

In future we plan to hold investigations related directly with experimental data in order to test mathematical model in practical usage.