

Space-Time Petrov-Galerkin Method for Fractional Diffusion Problems

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Abstract. We present and analyze a space-time Petrov-Galerkin finite element method for a time-fractional diffusion equation involving fractional derivative in time and zero initial data. We derive a proper weak formulation involving different solution and test spaces and show the inf-sup condition for the bi-linear form and thus its well-posedness. Further, we develop a novel finite element formulation, show the well-posedness of the discrete problem, and establish error bounds in both energy and least-squares norms for the finite element solution. In the proof of the discrete inf-sup condition, a certain nonstandard least-squares stability property of the L_2 projection operator plays a key role. We provide extensive numerical examples to verify the convergence of the method.