

A Hierarchical Finite Element Monte Carlo Method for Stochastic Two Scale Elliptic Problems

We consider two scale elliptic equations whose coefficient is random and depends on a macroscopic slow variable and a fast variable. We assume that the effective coefficient can be approximated by solving random cell problems in a finite size cube (this is the case, for example, of an ergodic random coefficient, or a random periodic coefficient). This approximated effective coefficient is, however, realization dependent; and we aim to compute its expectation. Straightforward employment of finite element approximation and the Monte Carlo method to compute this expectation with the same level of finite element resolution and the same number of Monte Carlo samples at every macroscopic point is prohibitively expensive. We develop a hierarchical finite element Monte Carlo algorithm to approximate the effective coefficients at a dense hierarchical network of macroscopic points. The method achieves an optimal level of complexity that is essentially equal to that for computing the effective coefficient at one macroscopic point, with essentially the same accuracy. The levels of accuracy for solving cell problems and for the Monte Carlo approximation are chosen according to the level in the hierarchy that the macroscopic points belong to. Solutions at those points at which the cell problems are solved with high accuracy and the number of samples in the Monte Carlo approximation is high are employed as correctors for the effective coefficient at those points at which the cell problems are solved with lower accuracy and fewer Monte Carlo samples are used.

The method combines the hierarchical finite element method for solving cell problems at a dense network of macroscopic points with the optimal complexity developed in D. L. Brown, Y. Efendiev and V. H. Hoang, with a hierarchical Monte Carlo algorithm that uses different number of samples at different macroscopic points depending on the level in the hierarchy that the macroscopic points belong to.