Hybrid two-step finite element method: flux approximation and recovery of primary variable [1]

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Abstract

We present a new two–step method based on the hybridization of mesh sizes in the traditional mixed finite element method. On a coarse mesh, the primary variable is approximated by a standard Galerkin method, whose computational cost is very low. Then, on a fine mesh, an $H(\mathrm{div})$ projection of the dual variable is sought as an accurate approximation for the flux variable.

Our method does not rely on the framework of traditional mixed formulations, the choice of pair of finite element spaces is, therefore, free from the requirement of inf-sup stability condition. More precisely, our method is formulated in a fully decoupled manner, still achieving an optimal error convergence order. This leads to a computational strategy much easier and wider to implement than the mixed finite element method. Additionally, the independently posed solution strategy allows to use different meshes as well as different discretization schemes in the calculation of the primary and flux variables.

We also present a method of improvement on the computed approximate solutions using iterative methods. The decrease in the error can be estimated and it can be used whether another iteration is needed. Also, a simple scheme will be introduced to obtain the approximate solutions for the primary variable from the flux approximation. The approximate solution for the primary variable has the same order of accuracy as the flux approximations. We provide numerical examples that conform the efficiency and convergence of our methods.

[1] This presentation is based on joint works with Imbumn Kim (Seoul), JeEun Ku (Oklahoma), and Young Ju Lee (Texas).