The Constraint Energy Minimizing Generalized Multiscale Finite Element Method

In this talk, we present Constraint Energy Minimizing Generalized Multiscale Finite Element Method (CEM-GMsFEM). The key part of the method is the design multiscale basis functions within GMsFEM framework such that the convergence of method is independent of the contrast and linearly decreases with respect to mesh size if oversampling size is appropriately chosen.

We would like to show a mesh-dependent convergence with a minimal number of basis functions. Our construction starts with an auxiliary multiscale space by solving local spectral problems. In auxiliary multiscale space, we select the basis functions that correspond to small (contrast-dependent) eigenvalues. These basis functions represent the channels (high-contrast features that connect the boundaries of the coarse block). Using the auxiliary space, we propose a constraint energy minimization to construct multiscale spaces. The minimization is performed in the oversampling domain, which is larger than the target coarse block. The constraints allow handling non-decaying components of the local minimizers. If the auxiliary space is correctly chosen, we show that the convergence rate is independent of the contrast (because the basis representing the channels are included in the auxiliary space) and is proportional to the coarse-mesh size (because the constrains handle non-decaying components of the local minimizers). The oversampling size weakly depends on the contrast as our analysis shows. The convergence theorem requires that channels are not aligned with the coarse edges, which hold in many applications, where the channels are oblique with respect to the coarse-mesh geometry. The numerical results confirm our theoretical results. In particular, we show that if the oversampling domain size is not sufficiently large, the errors are large. To remove the contrast-dependence of the oversampling size, we propose a modified construction for basis functions and present numerical results and the analysis. We will also present numerical results to show the performance.

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