Solution Methods of Optimal Complexity: from Sparse SPD Matrices to their Fractional Powers

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Hierarchical basis algebraic multilevel iteration (AMLI) methods for scalar elliptic problems are introduced at the beginning, assuming that the coefficient jumps are aligned with the coarsest triangulation. The derived relative condition number estimates are uniform with respect to mesh and coefficient anisotropy, coefficient jumps, and number of degrees of freedom [1]. More recently, robust multilevel methods for problems of high-frequency and high-contrast are developed. The advantages of nonlinear AMLI are discussed. The auxiliary space multigrid (ASMG) method for highly heterogeneous media, based on preconditioning of the weighted H(div)-norm is presented at the end [2].

The second part of the talk is motivated by the active on-going research in fractional diffusion problems. Several different techniques are proposed to localize the nonlocal operator, thus increasing the space dimension of the original computational domain. An alternative approach is developed in [3]. The main goal is to reduce the computational complexity. Let A be a properly scaled symmetric and positive definite (SPD) matrix. A method for solving algebraic systems of linear equations involving A^{α}, $0 < \alpha < 1$ is presented. The solver is based on best uniform rational approximations of the scalar functions $t^{\beta-\alpha}$, β is a small integer, Although the fractional power of A is a dense matrix, the algorithm has complexity of order O(N), where N is the number of unknowns. Some parallel scalability results are provided at the end [4].

References

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