

Hybrid dimension multiscale models for the flows in domains containing thin tube structures

Thin structures are some finite unions of thin rectangles (in 2D settings) or cylinders (in 3D settings) depending on small parameter $\varepsilon \ll 1$ that is, the ratio of the thickness of the rectangle (cylinder) to its length [1-4]. We consider thin structures and multistructures [5] which consist of several “massive” domains independent of ε connected by thin structures. Viscous flows in such structures are modeled by steady or non-steady Stokes or Navier-Stokes equations stated in thin structures or multistructures with the no-slip boundary condition at the lateral boundary of the cylinders and with the inflow and outflow conditions with the given velocity on some part of the boundary.

For thin structures an asymptotic expansion of the solution is constructed and justified. It has a form of a Poiseuille (or Womersley) flow within thin cylinders at some distance from the bases while the boundary layers near the ends of the cylinders decay exponentially. The algorithm of construction of the expansion deals with a special Reynolds type problem on the graph for the pressure. This structure of the expansion allows to reduce the dimension within the cylinders at the distance of order $\varepsilon |\ln \varepsilon|$ from the bases of the cylinders and derive the junction conditions between models of different dimensions. This approach is extended for multistructures (Stokes equations).

Finally, we discuss the possibility of asymptotic derivation of boundary conditions describing the elasticity of the wall and of non-Newtonian equations for the fluid motion.

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